

## Module - 02 Measure of Central Tendency

### AVERAGES:-

Central value or measure of central tendency.

According to Croxton and Cowdon.  
"An average value is a single value within the range of the data i.e. used to represent all the values in the series. Since, an average is somewhere within the range of the data, it is sometime called as measure of central value."

### MEANING OF AVERAGES:-

It is the value which lies between the smallest and the largest values of the observations in a distribution.

It is called call the measure of central tendency because <sup>it gives us</sup> an idea about the concentration of the value in the centre part of the distribution.

### OBJECTIVES:-

- \* To present the entire data in a single value which describes the characteristics of the entire data.

- \* To facilitates comparison
- \* To provide bases for decision making
- \* To obtain the clear picture of whole group (larger data) by the study of sample data.

### CHARACTERISTICS / ESSENTIALS / REQUISITES OF A GOOD AVERAGE:-

- \* It is essential or easy to understand [low complexity, Even layman should understand easily]
- \* It should not be affected by extreme points.
- \* It is simple to compute.
- \* It should be based on all observations.
- \* It is rigidly defined, the values are accurate, unbiased.
- \* It should be capable of further statistical treatments.

### VARIOUS TYPES OF MEASURE OF CENTRAL TENDENCY:-

- a) Arithmetic mean
  - (i) simple mean
  - (ii) weighted mean.
- b) Median
- c) Mode.
- d) Geometric mean
- e) Harmonic mean
- f) Percentiles
- g) Deciles, etc,

(a) ARITHMETIC MEAN :-

Arithmetic mean or mean is the number which is obtained by adding the values of all the items of a series and dividing the total by the number of items.

PROBLEMS OF ARITHMETIC MEAN:-  
[INDIVIDUALS SERIES / DIRECT METHOD]

1) Find out the mean marks of 10 students in corporate accounting

$x = 40, 48, 35, 59, 70, 92, 48, 68, 82, 28$

Soln

$\Sigma x = 570, N = 10$

$$\bar{x} = \frac{\Sigma x}{N} = \frac{570}{10} = \underline{\underline{57}}$$

2) Find out Arithmetic mean from the following data

$x = 10, 15, 20, 30, 50, 70, 80, 100$

Soln

$\Sigma x = 375$

$N = 8$

$$\bar{x} = \frac{\Sigma x}{N} = \frac{375}{8} = \underline{\underline{46.875}}$$

3) Find out arithmetic mean

$x = 10, 25, 40, 60, 80, 110, 140, 160, 180, 200$

Solve

$$\sum x = 1005$$

$$N = 10$$

$$\bar{x} = \frac{\sum x}{N} = \frac{1005}{10} = \underline{\underline{100.5}}$$

PROBLEMS ON DISCRETE SERIES:-

16 Following are the marks obtained by 50 B.com students. Calculate the Mean marks scored by these students.

x	f	fx
20	2	40
30	5	150
40	8	320
50	12	600
60	13	780
70	6	420
80	2	160
90	2	180
	50	2650

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{2650}{50}$$

$$= \underline{\underline{53}}$$

17 Find out AM from the following data

x	f	fx
5	8	40
5.5	15	82.5
6	35	210
6.5	20	130
7	10	70

7.5	2	15
	90	547.5

$$\bar{X} = \frac{\sum fx}{N} = \frac{547.5}{90} = \underline{\underline{6.0833}}$$

3b Find out AM from the following:-

X	f	fx
5	12	60
9	24	216
12	20	240
16	25	400
25	32	800
40	9	378
50	8	400
	130	2494

$$\bar{X} = \frac{\sum fx}{N} = \frac{2494}{130} = \underline{\underline{19.1846}}$$

4b Find out AM from the following:-

X	f	fx
5	4	20
10	9	80
15	20	300
20	10	200
25	6	150
30	5	150
35	9	315

40	2	80
45	1	45
	65	1340

$$\bar{x} = \frac{\sum fx}{N} = \frac{1340}{65} = 20.6153$$

4. From the following data compute AM:-

X	f	fx
200	20	4000
300	25	7500
400	32	12800
500	38	19000
600	20	12000
700	12	8400
800	8	6400
	155	70100

$$\bar{x} = \frac{\sum fx}{N} = \frac{70100}{155} = 452.2580$$

(e) Assumed Mean Method:-

X	f	d (100)	fd
200	20	0	0
300	25	100	2500
400	32	200	6400
500	38	300	11400
600	20	400	8000

700	12	500	6000
800	8	600	4800
	155		39100

(b)

x	f	d(500)	fd
200	20	-300	-6000
300	25	-200	-5000
400	32	-100	-3200
500	38	0	0
600	20	100	2000
700	12	200	2400
800	8	300	2400
	155		7400

$$\begin{aligned} \bar{x} &= A + \frac{\sum fd}{N} \\ &= 500 + \frac{7400}{155} = 500 + (-47.74) \\ &= 500 - 47.74 \\ &= 452.26 \end{aligned}$$

(a)

$$\begin{aligned} \bar{x} &= A + \frac{\sum fd}{N} \\ &= 200 + \frac{39100}{155} \\ &= 200 + 252.25 \\ &= 452.25 \end{aligned}$$

66 Findout AM from the following:-

x	f	d(x)	fd
5	4	0	0
10	8	5	40
15	20	10	200
20	10	15	150
25	6	20	120
30	5	25	125
35	9	30	270
40	2	35	70
45	1	40	40
	85		1015

$$\bar{X} = A + \frac{\sum fd}{N}$$

$$= 5 + \frac{1015}{85}$$

$$= 5 + 15.61$$

$$= \underline{\underline{20.615}}$$

### PROBLEMS ON CONTINUOUS SERIES:-

10 Findout the AM

x	f	M	fm
0-10	5	5	25
10-20	8	15	120
20-30	15	25	375
30-40	20	35	700



40-50	12	45	540
50-60	6	55	330
60-70	4	65	260
	70		2350

=> Direct Method

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{2350}{70} = \underline{\underline{33.5}}$$

=> Short cut Method

X	f	w	d(s)	fd
0-10	5	5	0	0
10-20	8	15	10	80
20-30	15	25	20	300
30-40	20	35	30	600
40-50	12	45	40	480
50-60	6	55	50	300
60-70	4	65	60	240
	70			2000

$$\bar{x} = A + \frac{\sum fd}{N}$$

$$= 5 + \frac{2000}{70}$$

$$= 5 + 28.57$$

$$= \underline{\underline{33.57}}$$

X	f	M	d(35)	fd
0-10	5	5	-30	-150
10-20	8	15	-20	-160
20-30	15	25	-10	-150
30-40	20	35	0	0
40-50	12	45	10	120
50-60	6	55	20	120
60-70	4	65	30	120
<del>70</del>	70			-100

$$\begin{aligned} \bar{X} &= A + \frac{\sum fd}{N} \\ &= 35 + \left( \frac{-100}{70} \right) \\ &= 35 - 1.43 \\ &= \underline{\underline{33.57}} \end{aligned}$$

Q. Find AM

X	f	M	fM
400-500	15	450	6750
500-600	35	550	19250
600-700	60	650	39000
700-800	34	750	25500
800-900	12	850	10200
900-1000	6	950	5700
	162		106400

$$\bar{X} = \frac{\sum fM}{N} = \frac{106400}{162} = \underline{\underline{658.79}}$$

= shortcut method.

C-I	f	n	d(A50)	fd
400-500	15	450	0	0
500-600	35	550	100	3500
600-700	60	650	200	12000
700-800	34	750	300	10200
800-900	12	850	400	4800
900-1000	6	950	500	3000
	162			33500

$$\bar{x} = A + \frac{\sum fd}{N}$$

$$= 450 + \frac{33500}{162}$$

$$= 450 + 206.79$$

$$= \underline{\underline{656.79}}$$

3b Find out AM under a direct method and shortcut method

x	f	n	fx
0-10	15	5	75
10-20	20	15	300
20-30	25	25	625
30-40	24	35	840
40-50	12	45	540
50-60	31	55	1705
60-70	71	65	4615
70-80	52	75	3900

80-90	25	85	2125
90-100	25	95	2375
	300		14100

=> direct method

$$\bar{x} = \frac{\sum fd}{N}$$

$$= \frac{14100}{300}$$

$$= \underline{\underline{57}}$$

=> shortcut method

x	f	m	d	fd
0-10	15	5	0	0
10-20	20	15	10	200
20-30	25	25	20	500
30-40	24	35	30	720
40-50	12	45	40	480
50-60	31	55	50	1550
60-70	71	65	60	4260
70-80	52	75	70	3640
80-90	25	85	80	2000
90-100	25	95	90	2250
	300			15600

$$\bar{x} = A + \frac{\sum fd}{N}$$

$$= 5 + \frac{15600}{300} = 5 + 52 = \underline{\underline{57}}$$

Ab Find AM

e-I	f	m	fm
100-200	12	150	1800
200-300	24	250	6000
300-400	33	350	11550
400-500	40	450	18000
500-600	50	550	27500
600-700	46	650	29900
700-800	32	750	24000
800-900	29	850	24650
900-1000	10	950	9500
	276		152900

∴ direct method

$$\bar{x} = \frac{\sum fm}{N}$$

$$= \frac{152900}{276}$$

$$= 553.98$$

$$= 554$$

∴ shortcut method.

e-I	f	m	d	fd
100-200	12	150	0	0
200-300	24	250	100	2400
300-400	33	350	200	6600
400-500	40	450	300	12000
500-600	50	550	400	20000

600-700	46	650	500	23000
700-800	32	750	600	19200
800-900	29	850	700	20300
900-1000	10	950	800	8000
	246			111500

$$\begin{aligned}\bar{x} &= A + \frac{\sum fd}{N} \\ &= 150 + \frac{111500}{246} \\ &= 150 + 403.98 \\ &= 553.98 \\ &\approx 554\end{aligned}$$

### STEP - DEVIATION :-

$$\bar{x} = A + \left[ \frac{\sum fd}{N} \times i \right]$$

$\bar{x}$  = Actual mean.

A = Assumed mean

f = frequency

N = Total no. of frequency

i = class interval

$$d = \frac{n - A}{i}$$

## PROBLEMS ON STEP-DEVIATION METHOD:-

10 Find AM

$x$	$f$	$m$	$d = \frac{m-A}{p}$	$fd$
0-10	5	5	-3	-15
10-20	8	15	-2	-16
20-30	15	25	-1	-15
30-40	20	35	0	0
40-50	12	45	1	12
50-60	6	55	2	12
60-70	4	65	3	12
	70			-10

$$\bar{x} = A + \left[ \frac{\sum fd \times i}{N} \right]$$

$$= 35 + \left[ \frac{-10}{70} \times 10 \right]$$

$$= 35 + (-1.43)$$

$$= 35 - 1.43$$

$$= \underline{\underline{33.57}}$$

26 Find AM

$x - I$	$f$	$m$	$d$	$fd$
400 - 500	15	450	0	0
500 - 600	35	550	1	35
600 - 700	60	650	2	120
700 - 800	34	750	3	102
800 - 900	12	850	4	48
900 - 1000	6	950	5	30
	162			335

$$\bar{x} = A + \left[ \frac{\sum fd \times i}{N} \right]$$

$$= 450 + \left[ \frac{335 \times 100}{162} \right]$$

$$= 450 + (206.79)$$

$$= \underline{\underline{656.79}}$$

38 Find AM

$x - I$	$f$	$m$	$d$	$fd$
0 - 10	15	5	0	0
10 - 20	20	15	1	20
20 - 30	25	25	2	50
30 - 40	24	35	3	72
40 - 50	12	45	4	48
50 - 60	8	55	5	40



80-90	41	65	6	406
70-80	50	75	7	364
60-70	25	85	8	200
50-60	25	95	9	225
	300			1560

$$\bar{Y} = A + \left[ \frac{\sum fd \times i}{N} \right]$$

$$= 5 + \left[ \frac{1560 \times 10}{300} \right]$$

$$= 5 + 52$$

$$= \underline{\underline{57}}$$

Ans Find AM

C-I	f	m	d	fd
100-200	12	150	-9	-48
200-300	24	250	-3	-72
300-400	33	350	-2	-66
400-500	40	450	-1	-40
500-600	50	550	0	0
600-700	46	650	1	46
700-800	32	750	2	64
800-900	29	850	3	87
900-1000	10	950	4	40
	246			11

$$\begin{aligned} \bar{X} &= A + \left[ \frac{\sum fd \times i}{N} \right] \\ &= 550 + \left[ \frac{11}{276} \times 100 \right] \\ &= 550 + 3.98 \\ &= \underline{\underline{553.98}} \end{aligned}$$

### WEIGHTED ARITHMETIC MEAN:-

weighted Arithmetic mean ~~necessaries~~ is the average calculated on the basis of weights or importances of items under consideration.

$$\bar{X}_w = \frac{\sum WX}{\sum W}$$

X = Variables

w = weights.

### PROBLEMS:-

14 A candidate obtain the following marks in BEU examination.

English-60, Kannada-50, Physics-80, Chemistry-70, ~~Maths~~ Mathematics-90, Biology-70. Admission to a science institute is based on weighted mean of the marks were weights for different

Subjects are given as:-

1, 1, 3, 3, 3, 2 respectively. Find out the weighted mean marks.

Soln

Subject	Marks (x)	weight (w)	wx
English	60	1	60
Kannada	50	1	50
Physics	80	3	240
Chemistry	70	3	210
Maths	90	3	270
Biology	70	2	140
		$\Sigma w = 13$	$\Sigma wx = 970$

$$\bar{x} = \frac{\Sigma wx}{\Sigma w} = \frac{970}{13} = 74.61$$

Qp A candidate obtained the following percentage of marks in different subjects in final exams.

English-46%, Statistics-68%, Accounts-47%, Management-54%, Auditing-75%. It is agreed to give double weights to the marks in statistics and management as compare to other subjects. Find out simple mean and weighted arithmetic mean.

Soln

Subject	Marks (x)	weight (w)	wx
English	46	1	46
Statistics	68	2	136
Accounts	71	1	71
Mgt	54	2	108
Auditing	75	1	75
	314	7	436

∴ direct method (simple mean)

$$\bar{X} = \frac{\sum x}{N}$$

$$= \frac{314}{7} = \underline{\underline{60.28}}$$

∴ weighted method.

$$\bar{X} = \frac{\sum wx}{\sum w}$$

$$= \frac{436}{7} = \underline{\underline{60.28}}$$

34 The following table gives the marks of 2 candidates. Find the weighted average marks of each candidates by which the 2nd candidate have to increase in subject B and all the marks remain the same in order that both the candidates should have same place.

Subjects	weights	x	y	w <sub>x</sub>	w <sub>y</sub>
A	1	70	80	70	80
B	2	65	64	130	128
C	3	58	56	174	168
D	4	63	60	252	240
	10			626	616

$$\bar{X} = \frac{\sum w_x}{\sum w}$$

$$= \frac{626}{10} = 62.6$$

$$\bar{Y} = \frac{\sum w_y}{\sum w}$$

$$= \frac{616}{10} = 61.6$$

Difference between two candidates

$$= 626 - 616$$

$$= 10 \text{ marks}$$

∴ candidate y should increase 10 marks in subject B, in order to be in the same place with candidate x.

116 Compute the weighted arithmetic mean of the index no from the data given belows-

Group	(x) Index no	(w) weight	wx
food	130	4	520
clothing	101	2	202
fuel	155	3	465
power	140	2	280
rent	170	3	510
Misc	180	2	360
		16	2337

$$\bar{x} = \frac{\sum wx}{\sum x}$$

$$= \frac{2337}{16} = 146.06$$

### COMBINED MEAN:-

When two or more arithmetic mean combined together to form a single arithmetic mean is called as combined mean.

$$\bar{x}_{1,2,3 \dots n} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2 + \dots + N_n \bar{x}_n}{N_1 + N_2 + \dots + N_n}$$

$\bar{x}_{1,2,3 \dots n} =$  combined mean

$N_1 =$  Total no. of items in the first group

$N_2 =$  Total no. of items in the second group

$N_3 =$  Total no. of items in the third group

$N_n =$  Total no. of items in the  $n^{\text{th}}$  group

$\bar{x}_1 =$  Arithmetic mean of first group

$\bar{x}_2 =$  Arithmetic mean of second group

$\bar{x}_n =$  Arithmetic mean of  $n^{\text{th}}$  group

### PROBLEMS:-

1. The arithmetic age of 1<sup>st</sup> group of 80 students is 10 years and 2<sup>nd</sup> group of 20 students is 15 years. Find out the combined mean age of all the students together.

Soln Given:  $N_1 = 80$   $N_2 = 20$ ,  $\bar{x}_1 = 10$   $\bar{x}_2 = 15$

$$\bar{x}_c = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{80(10) + 20(15)}{80 + 20}$$

$$= \frac{800 + 300}{100}$$

$$= \frac{1100}{100} = \underline{\underline{11 \text{ years.}}}$$

24 The number of students who took CA exam from 3 sections of a college is 70, 65, 60 respectively. The average marks scored by these students are 66, 68, 78.9 respectively. Find out the average of all the students combined together.

Soln Given:  $\bar{x}_1 = 66$ ,  $\bar{x}_2 = 68$ ,  $\bar{x}_3 = 78.9$ ,  
 $N_1 = 70$ ,  $N_2 = 65$ ,  $N_3 = 60$

$$\bar{X} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2 + N_3 \bar{x}_3}{N_1 + N_2 + N_3}$$

$$= \frac{70(66) + 65(68) + 60(78.9)}{70 + 65 + 60}$$

$$= \frac{4620 + 4420 + 4734}{195}$$

$$= \frac{13774}{195}$$

$$= \underline{\underline{70.63}}$$

34 The average income of 100 labourers is ₹20 if the average income of all 150 labourers is ₹25, calculate the average income of remaining 50 labourers.



Soln

$$N_1 = 100 \quad \bar{X}_1 = 20 \quad \bar{X}_C = 25$$

$$N_2 = 50 \quad \bar{X}_2 = ?$$

$$\bar{X}_C = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

$$25 = \frac{100(20) + 50(\bar{X}_2)}{150}$$

$$25 = \frac{2000 + 50\bar{X}_2}{150}$$

$$3750 = 2000 + 50\bar{X}_2$$

$$3750 - 2000 = 50\bar{X}_2$$

$$1750 = 50\bar{X}_2$$

$$\bar{X}_2 = \frac{1750}{50}$$

$$\therefore \boxed{\bar{X}_2 = 35}$$

Ans From the following information provided in the table, calculate

- (i) The average wages paid in J.P. Nagar.
- (ii) The average wages paid in Jayanagar.

work category	J.P. Nagar		Jayanagar	
	No. of workers	Avg. wages paid per wk	No. of workers	Avg. wages paid per wk
skilled workers	150	180	350	175
unskilled workers	850	130	650	125

Solve) The average wages paid in J.P Nagar.  
= calculation of average wage paid  
in J.P Nagar

$$\bar{x}_c = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{150(180) + 850(130)}{150 + 850}$$

$$= \frac{27000 + 110500}{1000} = \frac{137500}{1000}$$

$$= \frac{1375}{1000}$$

$$= \underline{\underline{1.375}}$$

(ii) calculation of average wage paid  
in Jayanagar.

$$\bar{x}_c = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{350(1.75) + 650(1.25)}{350 + 650}$$

$$= \frac{612.5 + 812.5}{1000}$$

$$= \frac{1425}{1000} = \underline{\underline{1.425}}$$

### MERITS OF ARITHMETIC MEAN:-

- \* It is simple to understand.
- \* It is easy to compute.
- \* It is based on all observations.
- \* It is helpful for further statistical, mathematical, and algebraic treatments.

### DEMERITS OF ARITHMETIC MEAN:-

- \* It is very much affected by extreme points.
- \* Arithmetic mean may lead to wrong conclusions if the details of the data are not correct or incorrect.
- \* It is not always reliable.

### MEDIAN:-

Median refers to the middle value in an arranged distribution. It is the positionable average which divides the series in the two equal parts.

### DEFINITION:-

According to L.R. Connor, "The median is that value of variable which divides the group into two equal parts, one part comprising all the values greater and the other all the values less than median."

### FORMULAE:-

⇒ Individual series

$$M_e = \text{Size of } \left[ \frac{N+1}{2} \right]^{\text{th}} \text{ item}$$

### PROBLEMS ON INDIVIDUAL SERIES:-

10 Find out median if

$$x = 6, 8, 7, 2, 0, 3, 11, 10, 1$$

Solut Ascending order.

$$0, 1, 2, 3, 6, 7, 8, 10, 11$$

$$\Rightarrow N = 9$$

$$M_e = \left[ \frac{N+1}{2} \right]^{\text{th}}$$

$$= \left[ \frac{9+1}{2} \right]^{\text{th}} = \left[ \frac{10}{2} \right]^{\text{th}} = \underline{\underline{5^{\text{th}} \text{ item}}}$$

$$\therefore \boxed{\text{Median} = 6}$$

11 Calculate the Median from the following

House no:-	1	2	3	4	5	6
family income	980	1020	720	800	1200	600
₹						
						2000

Soln-

$$N_1 = 7$$

600, 720, 800, 980, 1020, 1200, 2000

$$M_c = \left[ \frac{N+1}{2} \right]^{th}$$

$$= \left[ \frac{7+1}{2} \right] = \left[ \frac{8}{2} \right] = 4^{th} \text{ item.}$$

$\therefore 4^{th} \text{ item} = \underline{\underline{980}}$

38 The price of wheat per 40kgs in 10 different shops arranged in an array is ₹80, ₹85, ₹87, ₹90, ₹92, ₹95, ₹97, ₹100, ₹105 and ₹110, calculate the median price.

$$M_c = \left[ \frac{N+1}{2} \right]^{th} = \left[ \frac{10+1}{2} \right]^{th}$$

$$= \left[ \frac{11}{2} \right]^{th} = 5.5^{th} \text{ value.}$$

$$= \frac{5^{th} \text{ value} + 6^{th} \text{ value}}{2} \times 0.5 = \frac{92 + 95}{2}$$

$$= \frac{(95 - 92) \times 0.5}{2}$$

$$= \frac{5(0.5)}{2} = \frac{587}{2}$$

$$= \underline{\underline{293.5}}$$

44 From the following figures, calculate the median wages.

wages = 36, 32, 28, 22, 26, 20, 18, 38

18, 20, 22, 26, 28, 32, 36, 38

$$N = 8$$

$$M = \left[ \frac{N+1}{2} \right]^{th} = \left[ \frac{8+1}{2} \right]^{th} = \left[ \frac{9}{2} \right]^{th} = 4.5^{th} \text{ value}$$

$$\frac{26 + 28}{2} = \frac{54}{2} = \underline{\underline{27}}$$

$$\therefore \boxed{M = 27}$$

54 Find out median

x = 11, 13, 21, 28, 17, 14, 20, 18

11, 13, 14, 17, 18, 20, 21, 28 = 8 N = 8

$$M = \left[ \frac{N+1}{2} \right]^{th} = \left[ \frac{8+1}{2} \right]^{th} = \left[ \frac{9}{2} \right]^{th} = 4.5^{th} \text{ value.}$$

$$\Rightarrow \frac{17 + 18}{2} = \frac{35}{2} = \underline{\underline{17.5}}$$

$$\therefore \boxed{M = 17.5}$$

66 Determine the median

x	f	cf
10	2	2
20	3	5
30	4	9
40	6	15
50	10	25
60	5	30
70	2	32
	32	

$$Me = \left[ \frac{32+1}{2} \right]^{\text{th}} \text{ item}$$

$$= \left[ \frac{33}{2} \right]^{\text{th}} \text{ item}$$

$$= 16.5^{\text{th}} \text{ item}$$

∴ 16.5<sup>th</sup> item is 50

76 Findout median.

x	f	cf
1	7	7
2	11	18
3	15	33
4	20	53
5	10	63

6	5	68
7	1	69
	69	

$$M_e = \left[ \frac{69 + 1}{2} \right] = \left[ \frac{70}{2} \right]^{th} = 35^{th} \text{ item}$$

$\therefore 35^{th}$  item is A.

Ex: calculate median

Sizes (x)	f	cf
A5	5	5
A6	7	12
A7	9	21
A8	11	32
A9	10	42
S0	3	45
	45	

$$M_e = \left[ \frac{45 + 1}{2} \right]^{th}$$

$$= \left[ \frac{46}{2} \right]^{th}$$

$$= 23^{th} \text{ item}$$

$\therefore 23^{th}$  item is A8



96 10 coins were tossed together and the number of heads resulting was drawn, the tossing process was repeated 200 times and the following results were derived.

x	f	cf
0	3	3
1	9	12
2	16	28
3	24	52
4	46	98
5	37	135
6	25	160
7	21	181
8	12	193
9	5	198
10	2	200
	200	

$$M_e = \left[ \frac{200 + 1}{2} \right]^{th}$$

$$= \left[ \frac{201}{2} \right]^{th} \approx 100.5^{th} \text{ value.}$$

= 4 100.5<sup>th</sup> value is 5

## PROBLEMS ON CONTINUOUS SERIES:-

$M_e =$  Size of  $\left[\frac{N}{2}\right]^{\text{th}}$  item.

$$M_e = L + \left[ \frac{\frac{N}{2} - cf}{f} \right] \times i$$

$L =$  lower limit of the median class

$N =$  Total no. of frequencies

$cf =$  cumulative frequency of the class previous to median class

$f =$  Simple frequency of median class

$i =$  class interval.

1) calculate the median

c-I	f	cf
0-10	7	7
10-20	25	32
20-30	25	57
30-40	20	77
40-50	10	87
50-60	5	92
60-70	1	93
	83	

§  $\frac{N}{2} = \frac{93}{2} = 46.5^{\text{th}}$  item

$cf = 92, f = 5, i = 10, L = 50, N = 93$

$$M_c = L + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 20 + \left[ \frac{41.5 - 22}{25} \right] \times 10$$

$$= 20 + \left[ \frac{19.5}{25} \right] \times 10$$

$$= 20 + (0.78) \times 10$$

$$= 20 + 7.8$$

$$= \underline{\underline{27.8}}$$

∴ calculate the median

x	f	cf
0-5	5	5
5-10	10	15
10-15	13	28
15-20	12	40
20-25	7	47
25-30	3	50
	50	

$$\frac{N}{2} = \frac{50}{2} = 25^{\text{th}} \text{ item}$$

$$l = 10, f = 13, cf = 15, i = 5$$

$$M_c = l + \left[ \frac{N/2 - cf}{f} \right] \times p$$

$$= 10 + \left[ \frac{25 - 15}{13} \right] \times 5$$

$$= 10 + \left[ \frac{10}{13} \right] \times 5$$

$$= 10 + 3.846$$

$$= \underline{\underline{13.846}}$$

36 Findout median.

Marks	No. of students
80-90	8
70-80	13
60-70	23
50-60	37
40-50	25
30-40	12
20-30	7
10-20	4

Soln

The data is given in descending order, to findout median the series should be converted into ascending order.

Marks(x)	no. of students	cf
10-20	4	4
20-30	7	11
30-40	12	23
40-50	25	48
50-60	37	85
60-70	23	108
70-80	13	121
80-90	8	129
	129	

$$\frac{N}{2} = \frac{129}{2} = 64.5$$

$$l = 50, f = 37, cf = 48, i = 10$$

$$M_c = l + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 50 + \left[ \frac{64.5 - 48}{37} \right] \times 10$$

$$= 50 + \left[ \frac{16.5}{37} \right] \times 10$$

$$= 50 + 4.45$$

$$= \underline{\underline{54.45}}$$

*inclusive series*

Q6 Findout median

X	f
0-4	20
5-9	37
10-14	52
15-19	21
20-24	10

Solut.

X	f	cf
-0.5 - 4.5	20	20
4.5 - 9.5	37	57
9.5 - 14.5	52	109
14.5 - 19.5	21	130
19.5 - 24.5	10	140
	140	

∴ As the problem have inclusive series of the data, it has to be converted into exclusive series with the help of conversion factor.

$$\text{conversion factor} = \frac{\text{LL of under} - \text{UL of Previous}}{2}$$

$$= \frac{5 - 4}{2} = \frac{1}{2} = 0.5$$

∴ 0.5 - the conversion factor has to be deducted from the lower limit and

the same conversion factor should be added to the upper limit

$$\frac{N}{2} = \frac{140}{2} = \underline{\underline{70}}$$

$$l = 9.5, cf = 57, f = 52, i = 5$$

$$M = l + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 9.5 + \left[ \frac{70 - 57}{52} \right] \times 5$$

$$= 9.5 + \left[ \frac{13}{52} \right] \times 5$$

$$= 9.5 + 1.25$$

$$= \underline{\underline{10.75}}$$

54 Find out the median.

x	f
110-119	5
120-129	7
130-139	12
140-149	20
150-159	16
160-169	10
170-179	7

180-189      3

solut

x	f	cf
109.5 - 119.5	5	5
119.5 - 129.5	7	12
129.5 - 139.5	12	24
139.5 - 149.5	20	44
149.5 - 159.5	16	60
159.5 - 169.5	10	70
169.5 - 179.5	7	77
179.5 - 189.5	3	80
	80	

$$\frac{N}{2} = \frac{80}{2} = 40$$

$$l = 139.5, \quad cf = 24, \quad f = 20, \quad i = 10$$

$$M = l + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 139.5 + \left[ \frac{40 - 24}{20} \right] \times 10$$

$$= 139.5 + \left[ \frac{16}{20} \right] \times 10$$

$$= 139.5 + 8$$

$$= \underline{\underline{147.5}}$$



6\* Findout the median

Marks	cf
below 5	30
below 10	225
below 15	465
below 20	580
below 25	634
below 30	644
below 35	650
below 40	653
below 45	655

Soln:

c-I	f	cf
0-5	30	30
5-10	195	225
10-15	240	465
15-20	115	580
20-25	54	634
25-30	10	644
30-35	6	650
35-40	3	653
40-45	2	655
	655	

$$\frac{N}{2} = \frac{655}{2} = 327.5$$

$$l = 10, f = 240, cf = 225, h = 5$$

$$M_c = l + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 10 + \left[ \frac{122.5 - 205}{240} \right] \times 5$$

$$= 10 + \left[ \frac{102.5}{240} \right] \times 5$$

$$= 10 + 2.13$$

$$= 12.13$$

### MERITS OF MEDIAN:-

- \* It is easy to understand and calculate.
- \* It is not influenced by the magnitude of extreme deviations.
- \* It is the most appropriate average of quantitative data.
- \* It indicates the value of the middle item in the distributions.

### DEMERITS OF MEDIAN:-

- \* For calculating median it is necessary to arrange the data.
- \* It is a positional average, therefore each and every observation is not considered.
- \* As it ignores the extreme values, it may not always be the representative of the observations.

\* It is not capable of further mathematical and algebraic treatment

MODE OR MODAL VALUES-

Mode is that value in a series of observations which occurs with the greater frequency. The variable which repeats itself maximum number of time is called mode. It is denoted by  $Z$ .

According to A. M. Tuttle "Mode is the value which has greatest frequency or density, in its immediate neighbourhood."

The methods of calculating Mode are :-

- (i) Inspection method
- (ii) Graphic method
- (iii) Grouping and analysis method
- (iv) Empirical method

PROBLEMS ON MODE [inspection method]  
UNDER INDIVIDUAL SERIES:-

1) Findout the value of mode.

$X = 10, 12, 11, 10, 13, 14, 10, 14, 15, 16$

Soln:

X	Tally Bar	f
10		3
11		1
12		1
13		1
14		2
15		1
16		1
		10

∴ The value of Mode = 2 = 10, because it has highest frequency of 3

OR

∴  $z = 10$  because it has been repeated 3 times.

26 Find out the value of Mode

$x = 4, 8, 9, 12, 11, 8, 15, 8, 9$

Soln:

X	Tally Bar	f
4		1
5	-	0
6	-	0
7	-	0
8		3
9		2
10	-	0
11		1
12		1
13	-	0
14	-	0
15		1
		$N = 9$

∴  $z = 8$  because it has been repeated 3 times.

36 The marks obtained by 20 students in a class are given. Calculate Modal Marks. The marks are:-

24, 25, 30, 35, 25, 30, 37, 41, 40, 25, 20, 23, 45, 26, 25, 30, 40, 36, 22 and 21.

Solve

x	Tally Bar	f
20		1
21		1
22		1
23		1
24	-	0
25		4
26		1
27		1
28	-	0
29	-	0
30		3
31	-	0
32	-	0
33	-	0
34	-	0
35		1
36		1
37		1
38	-	0
39	-	0
40		2
41		1
42	-	0
43	-	0
44	-	0
45		1

$N=20$

$\therefore 2 = 25$  because it has been repeated 4 times.

Q. Find out of <sup>the</sup> value of Mode

$x = 12, 15, 18, 21, 18, 24, 24, 15$

Soln

$x$	Tally Bar	$f$
12		1
13	-	0
14	-	0
15		2
16	-	0
17	-	0
18		2
19	-	0
20	-	0
21		1
22	-	0
23	-	0
24		1
25	-	0
26	-	0
27		1
		$N = 8$

$\therefore$  Here, the mode is ill-defined because the variables 15 and 18, having the same amount of frequencies hence, it is a bimodal or multi-modal.

54 Find out the value of Mode:-

$x = 4, 8, 9, 10, 9, 11, 18, 17, 11$

x	Tally Bar	f
4		1
5	-	0
6	-	0
7	-	0
8		1
9		2
10	-	0
11		2
12		1
13	-	0
14	-	0
15	-	0
16	-	0
17		1
18		1
		$N=9$

∴ here, the mode is ill-defined because the variables 9 and 11, having the same amount of frequencies, hence, it is a bi-modal or multi-modal.

## PROBLEMS ON DISCRETE SERIES-

10 Find out the value of mode

X	f
1	10
2	15
3	35
4	55
5	10
6	7
7	10
N=142	

On the basis of inspection method the value of  $z$  is 4 because it has the highest frequency of 55.

11 Find out the value of mode

X	f
2	7
4	15
6	45
8	70
10	8
12	5
18	4
20	1
155	

On the basis of inspection method the value of  $z$  is 8 because it has the highest frequency is 70.



Q4 calculate the mode from-

x	1	2	3	4	5	6	7	8	9	10
f	3	5	11	18	26	23	17	10	4	2

∴ on the basis of inspection method the value of  $z$  is 5 because it has the highest frequency is 26.

Q5 calculate the mode from the following:-

Size of shoes (x)	No. of pairs sold (f)
1	11
2	5
3	12
4	7
5	12
6	18
7	30
8	36
9	10
10	9
$N = 150$	

∴ on the basis of inspection method the value of  $z$  is 8 because it has the highest frequency is 36.

## CALCULATION OF MODE UNDER CONTINUOUS SERIES:-

$$z = l + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times i$$

$z$  = mode

$l$  = lower limit of modal class

$i$  = class interval

$f_1$  = frequency of the modal class

$f_0$  = frequency of the class previous to the modal class

$f_2$  = frequency of the modal class next to the modal class.

### PROBLEMS:-

10 Calculate the mode from the following

$l-I$	$f$
5-10	16
10-15	34
15-20	96
20-25	33
25-30	17
$N = 196$	

On the basis of inspection method the modal value lies between 15-20

Soln  $l = 15$ ,  $f_1 = 96$ ,  $f_0 = 34$ ,  $f_2 = 33$   
 $i = 5$

$$z = l + \left( \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times i$$

$$= 15 + \left[ \frac{96 - 34}{2(96) - 34 - 33} \right] \times 5$$

$$= 15 + \left[ \frac{62}{125} \right] \times 5$$

$$= 15 + (0.496)5$$

$$= 15 + 2.48$$

$$= \underline{\underline{17.48}}$$

Q6 Calculate the mode from the following:-

C-I	f
10-15	2
15-20	4
20-25	9
25-30	13
30-35	15
35-40	8
40-45	6
	57

On the basis of inspection method the modal value lies b/w 30-35

$$l = 30, f_1 = 15, f_0 = 13, f_2 = 8, i = 5$$

$$Z = l + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times i$$

$$= 30 + \left[ \frac{15 - 13}{2(15) - 13 - 8} \right] \times 5$$

$$= 30 + \left[ \frac{2}{9} \right] \times 5$$

$$= 30 + 1.11$$

$$= \underline{\underline{31.1}}$$

3b Find out the value of  $z$

C-T	f
0-50	75
50-100	12
100-150	120
150-200	85
200-250	10
250-300	42
300-350	35
350-400	8

Soln

$$l = 100 \quad f_1 = 120 \quad f_0 = 12, \quad f_2 = 85 - 1 = 84$$

On the basis of ins

$$z = l + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times i$$

$$= 100 + \left[ \frac{120 - 12}{2(120) - 12 - 85} \right] \times 50$$

$$= 100 + \left[ \frac{108}{143} \right] \times 50$$

$$= 100 + 37.76$$

$$= \underline{\underline{137.76}}$$

44 Find out the value of z

x	f
0-9	12
10-19	8
20-29	14
30-39	7
40-49	25
50-59	38
60-69	12
70-79	3
80-89	2
90-99	6
	122

Soln

X	f
-0.5 - 9.5	12
9.5 - 19.5	8
19.5 - 29.5	14
29.5 - 39.5	7
39.5 - 49.5	25
49.5 - 59.5	33
59.5 - 69.5	12
69.5 - 79.5	3
79.5 - 89.5	2
89.5 - 99.5	6
	122

$$l = 49.5 \quad f_1 = 33 \quad f_0 = 25 \quad f_2 = 12 \quad h = 10$$

$$Z = l + \left[ \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

$$= 49.5 + \left[ \frac{33 - 25}{2(33) - 25 - 12} \right] \times 10$$

$$= 49.5 + \left[ \frac{8}{29} \right] \times 10$$

$$= 49.5 + 2.76$$

$$= \underline{\underline{52.76}}$$

## CALCULATION OF MODE UNDER EMPIRICAL FORMULA:-

⇒ When two or more items consists the maximum number of times or frequency we use empirical formula which is given by "Karl Pearson"

$$\text{Mode} = 3 \text{ median} - 2 \text{ mean.}$$

$$Z = 3M_e - 2\bar{x}$$

### PROBLEMS:-

Qp Find Z when  $\bar{x} = 21.76$  and  $M_e = 20.84$

Soln

$$Z = 3M_e - 2\bar{x}$$

$$= 3(20.84) - 2(21.76)$$

$$= 62.52 - 43.52$$

$$= 19$$

Qp If  $Z = 49$  and  $\bar{x} = 50.73$  find Median.

Soln

$$Z = 3M_e - 2\bar{x}$$

$$49 = 3M_e - 2(50.73)$$

$$49 = 3M_e - 101.46$$

$$49 + 101.46 = 3M_e$$

$$150.46 = 3M_e$$

$$\Rightarrow M_e = \frac{150.46}{3}$$

$$M_e = 50.1533$$

Q6  $\bar{x} = 20.2$ ,  $M = 22.1$  find  $Z$   
Sol  
 $Z = 3M - 2\bar{x}$   
 $= 3(22.1) - 2(20.2)$   
 $= 66.3 - 40.4$   
 $Z = 25.9$

Q7  $M_1 = 22$ ,  $Z = 20.5$  find  $\bar{x}$   
Sol  
 $Z = 3M - 2\bar{x}$   
 $20.5 = 3(22) - 2\bar{x}$   
 $20.5 = 66 - 2\bar{x}$   
 $20.5 - 66 = -2\bar{x}$   
 $+45.5 = +2\bar{x}$   
 $\bar{x} = 22.75$

MERITS OF MODE:-

- \* It is simple to calculate and in most cases it is located by inspection.
- \* It is not affected by extreme values.
- \* It is used to determine average of qualitative phenomenon which is expressed in numbers.
- \* It can also be determined graphically.

DEMERITS OF MODE:-

- \* It is not based on each and every item of the series.
- \* It is not defined, there are several formulas for determining mode, which gives different answers.



\* It is not capable of further algebraic treatment.

Thus, mode doesnot possess ~~many~~ <sup>many</sup> essentials of a good average but it is useful in qualitative aspects like average size of shoes, average number of accidents, average size of garments etc. which are expressed in numbers.

Q Findout AM, Median and Mode.

X = 5, 10, 15, 20, 25, 30, 35, 40

Soln Mean

$$\bar{x} = \frac{\sum x}{N}$$

$$= \frac{180}{8} = 22.5$$

$$\text{Median} = M = \left[ \frac{N+1}{2} \right]^{\text{th}} \text{ item}$$

$$= \left[ \frac{8+1}{2} \right] = \left[ \frac{9}{2} \right] \approx 4.5^{\text{th}} \text{ item.}$$

$$= 4 \left[ \frac{20+25}{2} \right] = \frac{45}{2} = 22.5$$

$$\begin{aligned} \text{Mode} &\approx 2 = 3M - 2\bar{x} \\ &= 3(22.5) - 2(22.5) \\ &= 67.5 - 45 \\ &= 22.5 \end{aligned}$$

Q6 Findout AM, Median and mode.

X	1	2	3	4	5	6	7	8	9	10	
f	11	5	12	7	12	18	30	36	10	9	150

Soln ⇒ Mean

X	f	fx	
1	11	11	$\bar{x} = \frac{\sum fx}{N}$ $= \frac{931}{150}$ $= \underline{\underline{6.20}}$
2	5	10	
3	12	36	
4	7	28	
5	12	60	
6	18	108	
7	30	210	
8	36	288	
9	10	90	
10	9	90	
	150	931	

⇒ Median

X	f	cf
1	11	11
2	5	16
3	12	28
4	7	35
5	12	47
6	18	65
7	30	95

8	36	131
9	10	141
10	9	150
	150	

$$M = \left[ \frac{150 + 1}{2} \right]^{\text{th}} \text{ item}$$

$$= \left[ \frac{151}{2} \right]^{\text{th}} = 75.5^{\text{th}} \text{ item}$$

$\therefore 75.5^{\text{th}}$  item is 7.

$\Rightarrow$  Mode :-

x	f
1	11
2	5
3	12
4	7
5	12
6	18
7	30
8	36
9	10
10	9

On the basis of inspection method, the value of  $Z = 8$  because it has the highest frequency of 36.

$$\begin{aligned} Z &= 3 \text{ median} - 2\bar{x} \\ &= 3M - 2\bar{x} \\ &= 3(7) - 2(6.20) \\ &= 21 - 12.42 \\ &= 8.58 \end{aligned}$$

36 Findout the value of mean, median, mode by using direct method and 2 through empirical formula.

C-I	f	m	fm	cf
100-110	4	105	420	4
110-120	6	115	690	10
120-130	20	125	2500	30
130-140	30	135	4050	60
140-150	30	145	4350	90
150-160	14	155	2635	104
160-170	8	165	1320	112
170-180	2	175	350	114
	114		16315	

$$\Rightarrow \text{Mean} = \bar{x} = \frac{\sum fm}{N}$$

$$= \frac{16315}{114} = 139.486 \approx 140$$

$\Rightarrow$  Median:-

$$\frac{N}{2} = \frac{114}{2} = 57$$

$$M_1 = l + \left[ \frac{N/2 - cf}{f} \right] \times i$$

$$= 130 + \left[ \frac{57 - 30}{30} \right] \times 10$$

$$= 130 + \left[ \frac{28.5}{30} \right] \times 10$$

$$= 130 + 9.5$$

$$= \underline{\underline{139.5}}$$

Mode

$$Z = 3M - 2\bar{X}$$

$$= 3(139.5) - 2(139.44)$$

$$= 418.5 - 278.88$$

$$= \underline{\underline{139.62}}$$

~~CS~~