BENGALURU CITY UNIVERSITY

II Semester B.Sc Degree Examination

Model Question Paper, Mathematics (core) under NEP

Model Paper-1

Time: 2Hours and 30 minutes

Max. Marks:60

Instructions: *Answer all the questions*

I Answer any SIX questions:

(6x2=12)

1. The binary operation * on the set of positive rational numbers, Q^+ is defined by a*b=$\frac{ab}{2}, \forall a,b \in Q^+$. Find identity element of Q^+ and the inverse of 3.
2. Find the number of generators of the cyclic group of order 60.
3. Define normal subgroup of a group.
4. If $\mathbf{f:(G,\bullet) \to (G',*)}$ is homomorphism then prove that **f(e)=** e', where e and e' are the identity elements of G and G' respectively.
5. Find $\frac{ds}{d\theta}$ for the curve $r = a(1-\cos\theta)$
6. Find the asymptotes parallel to the co-ordinate axes for the curve $xy^3 - x^2y - x^2 - 1 = 0$.
7. Evaluate $\int_0^{\frac{\pi}{2}} \cos^6 x\, dx$.
8. Find the length of the curve $4y^2 = x^3$ between **x=0 and x=5.**

II Answer any THREE questions:

(3x4=12)

9. Let (G,*) be a group and $a, b \in$ G, then prove that $(a*b)^{-1}=b^{-1}*a^{-1}$.
10. Show that $(z_6, \oplus_6)$ is a group, where Z_6=**{0,1,2,3,4,5}.**
11. In a group G prove that $o(a) = o(a^{-1}), \forall a,b \in G$.
12. Prove that every subgroup of a cyclic group is cyclic.
13. State and prove Fermat's theorem in groups.

III Answer any THREE questions:

(3x4=12)

14. Prove that a subgroup H of a group G is normal iff $gHg^{-1}=H, \forall g \in G$.
15. If $f:G \to G'$ is a homomorphism, then prove that f(G) is a sub group of G'.
16. Prove that the intersection of two normal subgroups is a normal subgroup.
17. Show that $\mathbf{f:G \to G'}$ defined by **f(x)=cosx + i sinx** is a homomorphism. Find ker(f).
18. State and prove Cayley's theorem.

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IV Answer any THREE questions: **(3x4=12)**

19. With the usual notation prove that $\tan\varphi = r\dfrac{d\theta}{dr}$.
20. Find the pedal form of the curve $r^n=a^n\cos n\theta$.
21. Find the envelop to the curve $\dfrac{x}{a}+\dfrac{y}{b}=1$ and **a + b = c**, where c is a parameter.
22. Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ is another cycloid.
23. With usual notation prove that the radius of curvature of the curve y=f(x) is

$$\rho=\frac{\left(1+\left(\dfrac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$$

V Answer any THREE questions: **(3x4=12)**

24. Evaluate $\displaystyle\int_0^{\pi} x\sin^7 x\,dx$
25. Find the area of the segment cut off from the parabola $y^2=2x$ by the straight line $y=4x-1$.
26. Find the surface of revolution of the asteroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ about the x-axis.
27. Find volume of solid obtained by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line.
28. Evaluate $\displaystyle\int_0^{a} x^3\sqrt{ax-x^2}\,dx$.

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Model Question Paper, Mathematics (core) under NEP

Model Paper-2

Time: 2Hours and 30 minutes **Max. Marks:60**

Instructions: *Answer all the questions*

I Answer any SIX questions: **(6x2=12)**

1. In the group of integers Z an operation * is defined by **a*b=a+b-3** $\forall a, b \in Z$. Find the inverse of 5.
2. Define a cyclic group.
3. Prove that every subgroup of an abelian group is normal.
4. Show that $\mathbf{f:(G,+) \to (G',\bullet)}$ **defined by** $\mathbf{f(x)=e^{x}}$, $\forall x \in G$ **is a homomorphism.**
5. Find the p-r equation of the cardioid $r=a(1-\cos\theta)$
6. Find $\frac{ds}{dx}$ for the curve $y^2=4ax$.
7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x\, dx$.
8. Rite the formula for surface of revolution of **y=f(x)** about the axis and the ordinates are **x=a and x=b.**

II Answer any THREE questions: **(3x4=12)**

9. Prove that $G=\{2^n / n \in Z\}$ is a group under multiplication.
10. Prove that a non-empty subset H of a group G is a subgroup of **G iff** $ab^{-1} \in H$, $\forall a,b \in H$.
11. G is a group and $a \in G$ be an element is of order n, prove that for any positive integer m, $a^m=e$ **iff** n is a divisor of m.
12. If H is a subgroup of a group G. Prove that there is a one to one correspondence between any to right cosets of H in G.
13. State and prove Lagrange's theorem in groups.

III Answer any THREE questions: **(3x4=12)**

14. Prove that a subgroup H of a group G is normal iff $ghg^{-1} \in H$, $\forall g \in G, h \in H$.
15. Let (Z,+) be the additive group of all integers and H={1,-1} be the multiplicative group.

    Define $f(z)=\begin{cases} 1, & \text{if z is even} \\ -1, & \text{if z is odd} \end{cases}$, show that f is a homomorphism.

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16. Prove that the product of to normal subgroups of a group is a subgroup of a group.
17. If $\mathbf{f:G \to G'}$ **is an homomorphism then prove that** (i) **f(e)=** $\mathbf{e'}$, where e and e' are identity elements in G and G' respectively and (ii) **f(a^{-1})=(f(a))$^{-1}$**, $\forall \mathbf{a} \in \mathbf{G}$.
18. State and prove fundamental theorem of homomorphism of groups.

IV Answer any THREE questions: **(3x4=12)**

19. Show that the curves $\frac{2a}{r} = 1 + \cos\theta$ and $\frac{2b}{r} = 1 - \cos\theta$ intersect orthogonally.
20. With usual notations prove that **p=r sin** φ and $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.
21. Prove that the evolute of the parabola $\mathbf{y^2=4ax}$ **is** $\mathbf{27ay^2=4(x-2a)^2}$.
22. Find all the asymptotes to the curve $\mathbf{x^3+2x^2y+xy^2-x^2-xy+2=0}$.
23. Find the radius of curvature of the cycloid **x=a(θ+sinθ), y=a(1+cosθ).**

V Answer any THREE questions: **(3x4=12)**

24. Find the reduction formula for $\int \sin^n x \, dx$, **n>0.**
25. Evaluate $\int_0^{\pi} x \sin^4 x \cos^2 x \, dx$.
26. Find the area of the cardioid **r=a(1+cosθ).**
27. Find length og the arc of the parabola $\mathbf{y^2=ax}$ cut off by the latus rectum.
28. Find the volume of the solid generated by revolving $\mathbf{r^2=a^2\cos2\theta}$ about the line $\theta = \frac{\pi}{2}$.

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Model Question Paper, Mathematics (core) under NEP

Model-3

Time: 2 Hours and 30 minutes **Max. Marks: 60**

Instructions: Answer all the questions

I Answer any SIX questions: **(6x2=12)**

1. Ina set R of real numbers , the operation, * is defined by $a*b=\frac{ab}{3}$ show that * is associative
2. Find all the left cosets of the subgroup $H=\{0,2,4\}$ of the group $Z_6\ w.r.t.\oplus_6$
3. Define quotient group of a group.
4. Prove that the homomorphic image of an abelian group is abelian.
5. Find the angle between radius vector and tangent for the curve $r=ae^{\theta\cot\alpha}$
6. Find radius of curvature at any point (p,r) on the curve $r^3=a^2p$
7. Evaluate $\int_0^{\pi/2}\sin^3 x\cos^2 x\,dx$
8. Write formula for volume of revolution obtained by revolving $y=f(x)$ between **x=a** **& x=b** about the **x-axis**

II Answer any three question **(3x4=12)**

9. Show that $(Z_7,\otimes_7)\ where\ Z_7=\{1,2,3,4,5,6\}$ is an abelian group
10. Show that a group G is abelian if and only if $(ab)^2=a^2b^2\quad\forall a,b\in G$
11. If the order of an element a of a group G is n and p is an integer prime to n then prove that a^p is also of order n
12. If G is a cyclic group pf order d and **G=<a>** then prove that $a^k\,(k<d)$ is also a generator of G if and only if $(k,d)=1$
13. Prove that any two right cosets of a subgroup h of a group G is either disjoint or identical

III Answer any three question **(3x4=12)**

14. Prove that a subgroup H of a group G is a normal subgroup of G if and only if every right coset of H in G is a left coset of H in G

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15. If $f : G \to G'$ is defined by $f(x) = -x \ \forall x \in G$ where G is additive group of integers, then prove that f is isomorphism and find Ker f
16. If H and K are two normal subgroup of G such that **H$\cap$K={e}** then show that every element of H commutes with every element of K
17. Let $f : G \to G'$ be a homomorphism of groups, with Kernel K then prove that f is one – one if and only if K={e} where 'e' is the identity element in G
18. Prove that every quotient group of a cyclic group is cyclic

IV Answer any three questions **(3x4=12)**

19. Show that the curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ cut orthogonally
20. Find pedal equation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$
21. With usual notations prove that the radius of curvature, $\rho = \dfrac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y} - \dot{y}\ddot{x}}$
22. Find evolute of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
23. Find all the asymptotes to the curve, $x^3 - 3xy^2 - 4y^3 - x + y + 3 = 0$

V Answer any three questions **(3x4=12)**

24. Find reduction formula for $\int \cos^n x\, dx,\ n > 0$
25. Evaluate $\int_0^\infty \dfrac{dx}{(1+x^2)^4}$
26. Find the length of one loop of the curve $3ay^2 = x(x-a)^2$
27. Find the area bounded between the cissoids $y^2(a-x) = x^3$ where a>0 and its asymptote
28. Find the surface area of the solid generated by revolving the catenary $y = c\cosh\left(\dfrac{x}{c}\right)$ from the vertex to any point (x, y) about the x-axis.

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