BENGALURU CITY UNIVERSITY

III SEMESTER B.Sc Degree Examination

Mathematics (Core) - Ordinary Differential Equations and Real Analysis – 1

Model Paper – 1

Time: 2 Hours and 30 Minutes

Max. marks: 60

I Answer any six questions $6 \times 2 = 12$

1. Show that the equation $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$ is exact.
2. Find the general solution of $(x^2 - 1)p^2 - 2xyp + y^2 - 1 = 0$.
3. Solve $(D^2 - 6D + 13)y = 0$.
4. Write the necessary and sufficient condition for the total differential equation Pdx + Qdy + Rdz = 0 to be integrable.
5. Test the nature of the sequence $\{1 + \cos n\pi\}$.
6. State the Cauchy's General Principle of convergence.
7. Test the convergence of the series $1^3 + 2^3 + 3^3 + \cdots + n^3$.
8. Define the nature of the geometric series

II Answer any three questions $3 \times 4 = 12$

9. Verify for exactness and solve $(ax + hy + g)\,dx + (hx + by + f)\,dy = 0$.
10. Solve $p^2 + 2pycotx - y^2 = 0$.
11. Solve $y = 2px - yp^2$.
12. Find the general and singular solution of $sinpx\ cosy = cospx\ siny + p$.
13. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$ where 'a' is the parameter.

III Answer any three questions $3 \times 4 = 12$

14. Solve $y'' + 3y' + 2y = cos^2x$.
15. Solve $x^2y'' - xy' + 2y = xlogx$.
16. Solve $y'' + y = tanx$ by using the method of variations of parameters.
17. Solve $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$
18. Verify the condition for integrability and solve $3x^2(y + z)dx + (z^2 + x^3)dy + (2yz + x^3) = 0$.

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

IV Answer any three questions $3 \times 4 = 12$

19. Examine the convergence of the sequences whose n^{th} term is

    (i) $\sqrt{n+1} - \sqrt{n}$ (ii) $\left(\frac{n+1}{n-1}\right)^n$

20. Prove that every convergent sequence is bounded.

21. Prove that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is (i) monotonically increasing (ii) bounded.

22. Discuss the nature of the sequence $\left\{x^{\frac{1}{n}}\right\}$, x > 0.

23. Find the limit of the sequence 0.5, 0.55, 0.555, ...

V Answer any three questions $3 \times 4 = 12$

24. Test the convergence of $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \cdots$

25. State and prove D'Alembert's Ratio Test for series of positive terms.

26. Discuss the convergence of the series $\sum_1^\infty \left(\frac{nx}{n+1}\right)^n$.

27. Examine the convergence, absolute & conditionally convergence of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$

28. Sum to infinity the series $\sum_1^\infty \frac{(n+1)(2n+1)}{(n+2)!}$

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$6 \times 4 = 12$

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Mathematics (Core) - Ordinary Differential Equations and Real Analysis – 1

Model Paper – 2

Time: 2 Hours and 30 Minutes Max. marks : 60

I Answer any six questions $6 \times 2 = 12$

1. Show that the equation $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ is exact.
2. Find the general solution of $(a^2 - x^2)p^2 + 2xyp + y^2 - b^2 = 0$.
3. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$.
4. Verify the condition for integrability for $2yzdx + zxdy - xy(1 + z)dz = 0$
5. Prove that $\left\{\frac{3n+4}{2n+1}\right\}$ is monotonically decreasing sequence.
6. If $\lim_{n\to\infty} a_n = l$ *then prove that* $\lim_{n\to\infty} |a_n| = |l|$.
7. Test the convergence of the series $\sqrt{\frac{1}{4}} + \sqrt{\frac{2}{6}} + \sqrt{\frac{3}{8}} + \cdots \sqrt{\frac{n}{2n+2}} + \cdots$
8. State the Rabee's Test for series of positive terms.

II Answer any three questions $3 \times 4 = 12$

9. Verify for exactness and solve $(x^2 - ay)\,dx + (y^2 - ax)\,dy = 0$.
10. Solve $6p^3 - 7p^2 - p + 2 = 0$.
11. Solve $x = y + p^2$.
12. Find the general and singular solution of $(y - px)(p - 1) = p$.
13. Show that the family of curves $\frac{x^2}{c} + \frac{y^2}{c+1} = 1$ is self-orthogonal where 'c' is the parameter.

III Answer any three questions $3 \times 4 = 12$

14. Solve $(D^2 - 2D + 4)y = e^x cosx$.
15. Solve $4x^2y'' + 4xy' - y = 4x^2$.
16. Solve $y'' + 9y = sec3x$ by using the method of variations of parameters.
17. Solve $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = x - y$
18. Verify the condition for integrability and solve $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$.

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IV Answer any three questions $3 \times 4 = 12$

19. Test the convergence of the sequences whose n^{th} term is

(i) $\frac{2n^2+3n+5}{n+3}\sin\left(\frac{\pi}{n}\right)$ (ii) $\frac{n+(-1)^n}{n}$

20. If $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ *then show that* $\lim_{n\to\infty}(a_n . b_n) = a.b$

21. Prove that a monotonically increasing sequence which is bounded above is convergent..

22. Discuss the nature of the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$.

23. Show that the sequence $\{x_n\}$ where $x_1 = 1$, $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2, $\forall n \geq 2$.

V Answer any three questions $3 \times 4 = 12$

24. Test the convergence of $\sum \frac{1.2.3....n}{3.5.7.9....(2n+1)}$

25. Discuss the nature of the series $\sum_1^\infty \frac{1}{n^p}$.

26. Discuss the convergence of the series $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$

27. Examine the convergence of the series $\frac{x}{\sqrt{1}} - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots$

28. Sum to Infinity the series $\frac{1}{2} + \frac{1}{3.2^2} + \frac{1}{5.2^3} + \frac{1}{7.2^4} + \cdots$

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3 × 4 = 12

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III SEMESTER B.Sc Degree Examination

Mathematics (Core) - Ordinary Differential Equations and Real Analysis – 1

Model Paper – 3

Time: 2 Hours and 30 Minutes

Max. marks : 60

I Answer any six questions $6 \times 2 = 12$

1. Show that the equation $(sinxcosy + e^{2x})dx + (cosxsiny + tany)dy = 0$ is exact.
2. Solve $p^2 - 5p + 6 = 0$ where $p = \frac{dy}{dx}$
3. Find the particular integral of $\frac{d^2y}{dx^2} + y = sin3x$.
4. Verify the condition for integrability for $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$
5. Define a convergent sequence with an example.
6. Discuss the boundedness of the sequence whose nth term is $(-1)^n \frac{1}{n}$.
7. Test the convergence of the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$
8. State the Leibnitz's Test for an alternating series.

II Answer any three questions $3 \times 4 = 12$

9. Verify for exactness and solve $(x^2 - 2xy + 3y^2)\,dx + (y^2 + 6xy - x^2)\,dy = 0$.
10. Solve $xp^2 + (y - x)p - y = 0$.
11. Solve $p^3 - 4xyp + 8y^2 = 0$.
12. Find the general and singular solution of $x^2(y - px) = p^2y$ using the substitution $x^2 = u$ and $y^2 = v$.
13. Find the orthogonal trajectories of the family of curves $r = a(1 - cos\theta)$.

III Answer any three questions $3 \times 4 = 12$

14. Solve $y'' + 2y' + 4y = e^x sinx$.
15. Solve $(x + 2)^2y'' - (x + 2)y' + y = 3x + 4$.
16. Solve $y'' - y = \frac{2}{1+e^x}$ by using the method of variations of parameters.
17. Solve $\frac{dx}{dt} + 2y = -sint$, $\frac{dy}{dt} - 2x = cost$
18. Verify the condition for integrability and solve $(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$.

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IV Answer any three questions $3 \times 4 = 12$

19. Discuss the convergence of the sequences whose n^{th} term is

    (i) $\sqrt{n^2+1}-1$ (ii) $\frac{3+7+11+\ +(4n-1)}{2n^2+3n}$

20. If $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$ *then show that* $\lim_{n\to\infty}(a_n + b_n) = a + b$

21. Prove that the limit of convergent sequence is unique.

22. Discuss the nature of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

23. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots\frac{1}{n!}$ is convergent.

V Answer any three questions $3 \times 4 = 12$

24. Examine the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots$

25. State and prove Cauchy's root test for the convergence of the series of positive terms.

26. Discuss the convergence of the series $\sum_1^\infty \sqrt{\frac{n+1}{n^3+1}}x^n$

27. Test the convergence of the series $1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\cdots \quad (p>0)$

28. Sum to infinity the series $1+\frac{2}{6}+\frac{2.5}{6.12}+\frac{2.5.8}{6.12.18}+\cdots$

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