K.L.E Society's
S. Nijalingappa College
II BLOCK RAJAJINAGAR, BENGALURU -10

PG Department of Mathematics

QUESTION BANK

Algebra-II

1. Define Nilpotent element. Prove that the Nil radical of the ring A is equal to the Intersection of all prime ideal of A.
2. Define Jacobson Radical. For $x \in$ J(A), iff (1-xy) $\forall y \in A$.
3. For any ring A, N (A[x]) = J (A[x]).

4. If I_1, I_2 , ………I_k be a ideal of a ring which are pairwise co-prime. Hence Prove $\prod_{k=1}^{n} I_k = \bigcap_{k=1}^{n} I_k$

5. For any ideal I and J of A
   i. $I \subseteq r(I)$ and $I \subseteq J$ $\Longrightarrow r(I) \subseteq r(J)$
   ii. $r(r(I) = r(I)$
   iii. $r(IJ) = r(I \cap J) = r(I) \cap r(J)$
   iv. $r(I) = 1$ iff $I = (1)$
   v. $r(I + J) = r(r(I) + r(J))$
6. Define contraction of Ideals. Let f from $f: a \longrightarrow b$ be a ring homomorphism. Let I and J be ideals of Aand B respectively.
   i $I^e = I^{ece}$, $J^C = J^{cec}$
   ii If C denotes the set of contracted ideals of A and E denotes the set of all extended ideal of B . C={ I / $I^{ec} = I$} and E={ I / $J^{ce} = J$} . Further the map $I \longrightarrow I^e$is abijection map of C into E whose inverse is $J \longrightarrow J^c$.
7. Define Sub module of a module. State and Prove 3rd Isomorphism theorem.
8. State and Prove Nakayama's Lemma.
9. Define Jacobson Radical J(A) of a commutative ring A. Prove that For $x \in$ J(A),

   iff (1-xy) $\forall x \in A$.
10. Define Notherian and Artinian Module. Prove that suppose M is Notherian A-Module.

    Let N be a submodule of M then M is Noetherianiff N and M/N is Noetherian
11. State and Prove Nakayama's Lemma.

    Define D-ring Endomorphism. And Hence State and prove Schur's Lemma.
12. Define Sub module of a module. State and Prove 3rd Isomorphism theorem
13. Define exact sequence of Modules. If $O \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow O$ is a exact sequence of

    A-modules then show that M is Noetherian if and only if $M' and\ M''$ are Noetherian.
14. Show that submodule of finitely generated module need not be finitely generated.

15. Define contraction of Ideals. If C denotes the set of contracted ideals of A and E denotes the set of all extended ideal of B . C= { I / $I^{ec} = I\}$ and E={ I / $J^{ce} = J\}$. Further the map $I \longrightarrow I^e$ is a bijection map of C into E whose inverse is $J \longrightarrow J^c$.

16. The element $a \in K$ is algebraic over F iffF(a) is Finite extension of F.
17. Define algebraic element of a filed. And hence prove that

    $i)$ $a = \sqrt{2},\ b = \sqrt{3}$are algebraic of degree over Q

    ii) $Q(\sqrt{2}, \sqrt{3}) = Q(\sqrt{2} + \sqrt{3})$.

    iii) $Q(\sqrt{2} + \sqrt{3})$is algebraic extension of Q of degree 4
18. Let $f(x) \in F(x)$ be a degree of n≥1. Then prove that there is an extension E of F of degree at most n! in which f(x) has n-Roots.
19. Prove that [L: F] = [L: K] [K:F].
20. If $a \in K$ is algebraic of degree n then prove that [F(a): F]=n, and solve $a \pm b, ab\ and\ a/b$ are all algebraic over F.
21. Define D-ring Endomorphism. And Hence State and prove Schur's Lemma.
22. Define Notherian and Artinian Module.an A Module M is Noetheriniff every submodule is finitly generated
23. It is impossible to construct the Septagon by using compass and straight Edge.
24. It is possible to construct the Pentagon by using compass and a straight edge.
25. It is impossible to trisect $60^0$by using compass and a straight edge.