K.L.E Society's
S. Nijalingappa College
II BLOCK RAJAJINAGAR, BENGALURU -10

PG Department of Mathematics
QUESTION BANK

Algebra [M101T]

7 Marks

1. Define dihedral group. And show that D_8 group is not cyclic with Caylee's table.
2. State and Prove Calyles Theorem.
3. If a permutation is a product of S transpositions and also a product of T transpositions. Show that both S and T even and odd.
4. Verify the class equation for D3.
5. State and prove fundamental theorem of Homomorphism
6. Let G be a group acting on set S and x∈S then stab(x) is a subgroup of G.
7. Define Automorphism. If N and M are two normal subgroups of G then prove that $\frac{NM}{M} \approx \frac{N}{N \cap M}$
8. State and prove second Isomorphic Theorem
9. Define a principal ideal, principal ideal ring and show that the set of integers is a PIR.
10. Define a maximal ideal, prime ideal and show that in a commutative ring with unity, a maximal ideal is a prime ideal.
11. Define a Euclidean ring and show that the set of Gaussian integers is a Euclidean ring.
12. Let R be an integral domain with ideal P, then show that P is a prime ideal if and only if $\frac{R}{P}$ is an integral domain.
13. Show that in a PIR, every non-zero prime ideal is maximal.
14. State and prove first Isomorphic Theorem.
15. .Define Alternating group hence Prove that Sn/An = { 1, -1}
16. Define stabilizer of a group. State and prove Orbit-Stabilizer theorem.
17. Prove that If G be a finite set acting on S and K denote the number of orbits in S then
    a. K=$\frac{1}{O(G)} \sum_{g \in G} O(S_g)$

18. Let G be a group, an element a in G solve the class equation of Order of G by Using Ca $=\frac{O(G)}{O(Na)}$.
19. Define Solvable group. A group G Iff the $G^n = \{e\}$ for $n \geq 1$.
20. A Normal H of a group is maximal iff G/H is simple
21. State and Prove Cauchy's Theorem for Syllow's group.
22. If A and B are finite order subgroups of a group G then show that $O(AxB) = \frac{O(A).O(B)}{O(A \cap xBx^{-1})}$

10 Marks

1. State and Prove Syllow's 3^{rd} Theorem.
2. State and prove Syllow's First theorem.
3. Define a Simple group. Show that the A60 is simple.
4. Define Composition series. State and prove Jorden Holder Theorem.
5. State and prove Jorden Holder Theorem.

Topology [M303T]

7Marks

1. Define Cauchy's Sequence. Let X be a complete space and Y be a subspace of X the Y is complete iff it is closed
2. Define closure of a set. In a topological space (X, τ) for any A⊆X. Show that A∪d(A) is closed.
3. Define closure of a set. In a topological space (X, τ) for any A⊆X. show that A∪d(A) is closed.
4. Define Interior of a set. For any A⊆X, show that AO = $(\bar{A}1)1$
5. Define the continuity of a function in topological space. $f : \rightarrow Y$ is continuous at x iff V is a neighbourhood of f(x) ⇒ f^{-1}(V) is nbd of x.
6. State and prove pasting lemma.
7. Define Homeomorphism. A bijective function $f:X \rightarrow Y$ is a Homeomorphism, iff f($\bar{A}$) ⊆ $f(A\,\bar{)}$ for any set A⊆X.
8. Define topological Space. In topological space (X, τ) show that
   i. Arbitrary intersection of closed sets is closed.
   ii. Finite Union of closed sets is closed.

10-Marks

1. State and prove contraction mapping theorem.
2. State and prove Schroder- Bernstein Theorem
3. State and prove Bair's category theorem
4. State and prove cantor's intersection theorem.
5. Define bases of a topological space with an example. And Prove that is subfamily β of τ is a base for τ iff U∈τ and x∈U implies there exists B∈ β such that x∈B⊆U.

6. Let (X,τ) be any topological space, then for any set $A,B \subseteq X$. Show that
   i. $d(\Phi)=\Phi$
   ii. $A\subseteq B \Rightarrow d(A) \subseteq d(B)$
   iii. $d(A\cup B) = d(A)\cup d(B)$

7. Define limit point of a topological space. If $X =\{ a, b, c \}$,$\tau =\{ \Phi, X, \{a\},\{b, c\} \}$
   i. If $A=\{c\}$ find d (A).
   ii. Let $A\subseteq(X,d)$ then followings are equivalent
      i) X-A is Open
      ii) $d(A) \subseteq A$ Where d(A) is the set of all limit points of A