K.L.E Society's
S. Nijalingappa College

II BLOCK RAJAJINAGAR, BENGALURU -10

## PG Department of Mathematics QUESTION BANK

## Discrete Mathematics [ M303T]

7-MARKS

1. $A=\{a, b, c, d\}$
$R=\{(a, d),(c . d),(b . a),(b, c),(d, c)\}$
$\mathrm{S}=\{(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{b}),(\mathrm{c}, \mathrm{c}),(\mathrm{d}, \mathrm{d})\}$
Find $R{ }^{\circ} S, S{ }^{\circ} R, R^{2}, S^{2}$
2. Using the Warshall's algorithm, find the transitive closure of the relation ' $R$ ' defined on a set $A=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\} \& \mathrm{R}=\{(\mathrm{a}, \mathrm{d}),(\mathrm{c}, \mathrm{d}),(\mathrm{c}, \mathrm{a}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{c}),(\mathrm{d}, \mathrm{c})\}$
3. Using the Warshall's algorithm, find the transitive closure of the relation ' $R$ ' defined on a set $A=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\} \& \mathrm{R}=\{(\mathrm{b}, \mathrm{e}),(\mathrm{c}, \mathrm{b}),(\mathrm{c}, \mathrm{e}),(\mathrm{d}, \mathrm{a}),(\mathrm{e}, \mathrm{b}),(\mathrm{e}, \mathrm{c})\}$
4. If R is a relation on $\mathrm{A} \&|\mathrm{~A}|=\mathrm{n}$ then $R^{\infty}=R \cup R^{2} \cup R^{3} \cup \ldots . R^{n}$.
5. If $\mathrm{R} \& \mathrm{~S}$ are two relations define a given set A then prove that $(R \cap S)^{2} \subseteq R^{2} \cap S^{2}$.
6. Given a set $A$ with $|A|=n$ and a relation $R$ on $A$, let $M$ denote the matrix of $R$, then prove the following :
i. $\quad \mathrm{R}$ is reflexive if and only if $I_{n} \leq M$,
ii. $\quad \mathrm{R}$ is transitive if and only if $M^{2} \leq M$,
iii. $\quad \mathrm{R}$ is antisymmetric if and only if $M \cap M^{T} \leq I_{n}$ where $I_{n}$ is the unit matrix and $M^{T}$ is the transpose of M .
7. Define Poset \& Lattice .Draw Hasse digraph for $\left\langle D_{30}, /\right\rangle$.
8. Define a distributive lattice \& complemented lattice .In a distributive lattice, if an element has a complement then prove that this complement is unique.
9. In Boolean algebra for any two elements $\mathrm{a} \& \mathrm{~b} \& \mathrm{a}=\mathrm{b}$ iff $(a \wedge \bar{b}) \vee(\bar{a} \wedge b)=0$
10. For any $\mathrm{a}, \mathrm{b}, \mathrm{c}$ in a lattice $(\mathrm{A}, \preccurlyeq)$ prove
i. $\quad a \vee(b \vee c)=(a \vee b) \vee c$
ii. $\quad a \wedge(b \wedge c)=(a \wedge b) \wedge c$
11. State and prove Demorgan's law of Boolean algebra.
12. Find the generating function of the sequence

- $\{1,1,1,------\}$
- $\{0,1,2,3------\}$
- $\left\{1^{3}, 2^{3}, 3^{3}---\right\}$

13. In how many ways we can distribute 8 identical chocolates among 3 distinct children if each receives atleast 2 chocolate but not more than 4 .
14. Find the number of ways in which four of the letters of the word ENGINE be arranged by using exponential generating function.
15. Solve the recurrence relation

- $a_{n}+a_{n-1}-6 a_{n-2}=0$
- $a_{n}=3 a_{n-1}-2 a_{n-2}$ with $a_{1}=5, a_{2}=3$

16. Solve recurrence relation using generating function $a_{n}=3 a_{n-1}+2, \quad a_{0}=1$
17. Write a short on modeling "The Tower of Hanoi Problem" and solve it explicitly.
18. Write a short note on Rabbit population problem as a recurrence relation and solve it explicitly.
19. Define planar and non-planar graph .give an example for each
20. State and prove Euler's polyhedron formula.
21. If G is a plane graph in which every face is bounded by n-edges or n-cycle then $q \leq$ $\frac{n(n-1)}{n-2}$
22. The complete graph $K_{5}$ and the complete bipartite graph $K_{3,3}$ are non-planar.
23. For any graph G, $K(G) \leq \lambda(G) \leq \delta(G)$.
