



PG Department of Mathematics
QUESTION BANK

Elementary number Theory:

- 1 State well ordering principle and hence prove the division algorithm
- 2 State and prove Euclidean Algorithm
- 3 Define relatively prime integers and hence prove that two integers a and b with atleast one of them is different from zero are relatively prime if and only if there exist integers x and y such that $1 = ax + by$
- 4 Find the GCD between 1745 and 1485.
- 5 Prove that if $k > 0$ then $\gcd(ka, kb) = \gcd(a, b)$.
- 6 State and prove fundamental theorem of arithmetic
- 7 Is the Linear Diophantine Equation $33x + 14y = 115$ can be solved ?
- 8 Find the value of x and y such that $172x + 20y = 1000$.
- 9 a) Define followings
 - i. Quadratic residue.
 - ii. Legendre symbol
 - iii. Jacobi symbol
- 10 State and Prove Gauss Lemma
- 11 State and prove Euler-Fermat Theorem.
- 12 State and prove Chinese remainder theorem
- 13 State and prove Wolstenholme's Theorem.

14 Show that the following set are reduced residue system $\{1,5,7,11\}$ under mod 15.

15 State and prove Fermats Last Theorem

16 State well ordering principle and hence prove the division algorithm

17 Define relatively prime integers and hence prove that two integers a and b with atleastone of them is different from zero ,the for a positive integer d then $d = \gcd(a,b)$ iff

i. $d|a$ and $d|b$

ii. whenever $c|a$ and $c|b$ the $c|d$

18 Prove that for positive integer a and b , $\gcd(a,b) \operatorname{lcm}(a,b) = ab$.

19 Determine all the solutions in the positive of the integers of the Diophantine

Equation $1485x + 1745y = 15$

20 Define prime numbers. Use Euclidian algorithm to obtain integrs x and y for $\gcd(3024,12378)$

21 State and prove Fundamental Theorem of Arithmetic

22 Define Fermat and Mersenne Primes. Prove that there are infinitely primes.