K.L.E Society's
S. Nijalingappa College
II BLOCK RAJAJINAGAR, BENGALURU -10

PG Department of Mathematics
QUESTION BANK

Graph Theory

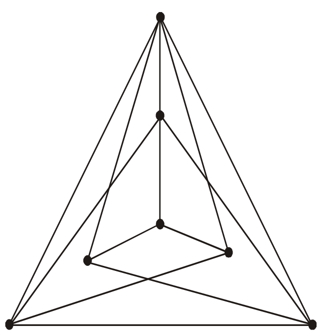
1. Prove that a vertex ϑ of a connected graph G is a cut-vertex of G if and only if there exists vertices u and ϑ distinct from v such that ϑ lies on every $u - w$ path of G.

2. Define a non-seperable graph with an example. Show that a graph of order at least three is non-seperable if and only if every two vertices lie on a common cycle.

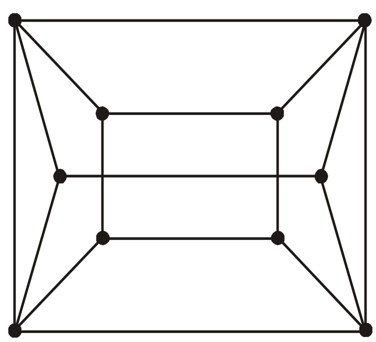
3. Define planar graphs. Let G be a connected planar graph with n-vertices, m-edges, r-regions, ρ-rank, μ-nullity and G^* be its geometric dual with n^*-vertices, m^*-edges, r^*-regions, ρ^*-rank, μ^*-nullity. Prove that $n = r^*, r = n^*, m = m^*, \rho = \mu^*, \mu = \rho^*$.

4. State and prove Euler's identity for connected planar graphs.
   Further verify the identity for the following graphs.

   (i)

   (ii)

5. (a) Define the following with an example.
   i. Chromatic number
   ii. Maximum independent set
   iii. Colour class

6. State and prove five colour theorem.

7. (a) Prove that for a graph of order

   'n' $\dfrac{n}{\beta(G)} \leq 1 + \Delta(G)$ for a graph G with

$\beta(G)$ – independence number of G.

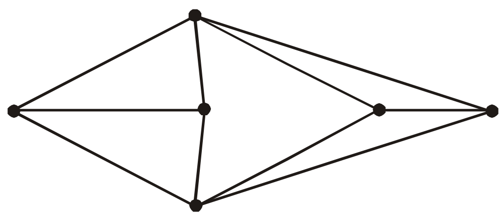
$\chi(G)$ – chromatic number of G.

$\Delta(G)$ – Maximum degree of G.

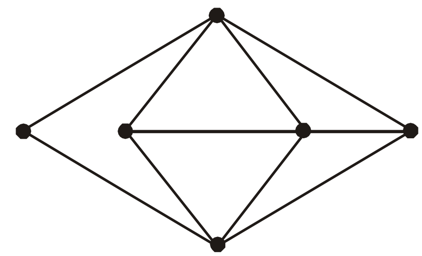
8. State and prove Decomposition theorem.

9. Use the theorem to establish that the chromatic polynomial of the following graphs are equal.

(i)



(ii)



10. Prove that a non-trivial connected digraph D is Eulerian if and only if outdegree of ϑ is equal to indegree of ϑ for all vertices ϑ of D.

11. Establish the following.

(i) A Tournment T is transitive if and only if T has no cycles.

(ii) If u is a vertex of maximum out degree in a tournament T then $\vec{d}(u,\vartheta) \leq 2$ for every vertex ϑ of T. **7**

12. Prove that for every graph G of order $'n'$ containing no isolated vertex then $\alpha_1(G) + \beta_1(G) = n$, where $\alpha_1(G)$ is the minimum edge cover and $\beta_1(G)$ is the cardinality of minimum edge independent set.

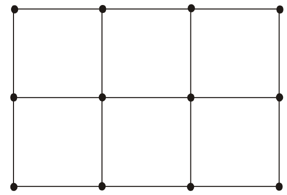
13. Prove that a graph G is two factorable if and only if G is r-regular for some positive even integer $'r'$.

14. State and prove "Havel-Hakimi" theorem. Further use the theorem to check whether the following sequences are graphical or not.

(i) 7, 5, 4,4,4,3,2,1

(ii) 7,5,3,6,4,4,2,1

15. Show that for the graph G in the figure the domination number is four.



16. Let G be a graph without isolated vertices. If S is a minimal domination set of G then, prove that $V(G) - S$ is a dominating set of G.

17. Define

(i) Edge connectivity of a graph G.

(ii) Block of a graph G with an example.

18. Show that a graph of order at least 3 is non-separable if and only if every 2 verticies lie on a common cycle.

19. State and prove Menger's Theorem.

20. If G is a connected plane graph of order n, size m and having regions r then prove that $n - m + r = 2$.

21. Explain the construction of Dual of a graph with an example.

22. Define an abstract dual of a graph. Prove that a graph G is planar if and only if it has to abstract dual.

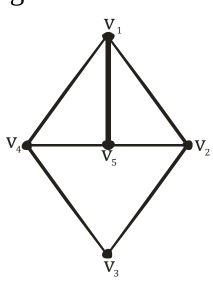
23. Define chromatic number of a graph.

24. What is the chromatic number of a complete graph K_n, cycle C_n and a bipartite graph $K_{m.n}$ where n is the order of the graph m, n are the orders of two partite sets of a bipartite graph.

25. Prove that a graph G is bipartite if and only if it does not contain odd cycle.

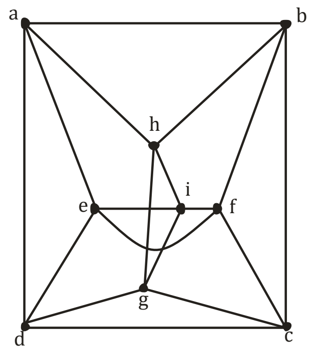
26. Let a and b be two non-adjacent verticies in a graph G. Let G' be a graph obtained by adding an edge between a and b and G'' be a simple graph obtained by fusing a and b together and replacing parallel edge with single edge then prove that $P_n(\lambda)$ of $G = P_n(\lambda)$ of $G' + P_{n-1}(\lambda)$ of G'' where $P_n(\lambda)$ be the chromatic polynomial for n vertices.

27. Using the above theorem obtain the chromatic polynomial of the following figure.



28. Prove that every planar graph is 5-colourable.

29. Determine the edge chromatic number of the following graph.



30. Write a short note on types of digraph.

31. Show that a non-trivial connected diagraph D is Eulerian if and only if out degree of v = indegree of v for all vertices v of D .

32. Define a Tournament.

Write all the possible tournaments of order 3 and order 4

33. Prove that a tournament T is transitive if and only if T has no cycles.

34. Let G be a bipartite graph with partite sets U and W such that $r = |U| \leq |W|$ then show that G contains a matching of cardinality r if and only if U is neighborly.

35. Define edge cover and edge independence number of a graph G.

36. For every graph G of order n containing no isolated vertices prove that $\alpha_1(G) + \beta_1(G) = n$ where $\alpha_1(G)$ is edge covering number and $\beta_1(G)$ is edge independence number.

37. Prove that every 3-regular bridgeless graph contains 1-factor.

38. State and prove Havel-Hakimi theorem.

39. Define dominating set of a graph. Show that the $\gamma(C_n) = \lceil n/3 \rceil$ for $n \geq 3$ where $\gamma(C_n)$ is the dominating number of a cycle of order n.

40. Using Havel-Hakimi algorithm check whether the following sequence is graphical or not.

(i) 5, 4, 3, 3, 2, 2, 2, 1, 1, 1.

(ii) 7, 7, 4, 3, 3, 3, 2, 1

41. What will be the domination number of the following graphs.

(i) Complete graph – K_n

(ii) Bipartite graph – $K_{s,t}$

(iii)Petersen's Graph.