BENGALURU CITY UNIVERSITY

I SEMESTER B.Sc. MATHEMATICS(CORE)

MODEL QUESTION PAPER - 1(2021-22 onwards) NEP

Max Marks: 60 Time: 3hrs

I. Answer any SIX questions. **(6x2=12)**

1. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
2. Define Rank of a matrix.
3. Find the n^{th} derivative of cos^2x.
4. Prove that every differentiable function is a continuous function.
5. If z = x^3- 3xy^2 then show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
6. Evaluate $\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)$
7. Verify Lagrange's mean value theorem for f(x) = (x-1)(x-2) in [0,4]
8. Prove that there is a minimum at (0,0) for the function f(x)= $x^3 + y^3$ -3xy

II. Answer any THREE questions. **(3x4=12)**

9. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ by reducing it to row reduced echelon form.
10. Find the real value of λ for which the system the system has non-zero solution

$$(1-\lambda)x+2y+3z=0$$
$$3x+(1-\lambda)y+2z=0$$
$$2x+3y+(1-\lambda)z=0$$

11. Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A=\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$

12. State and prove Cayley-Hamilton theorem.
13. If $A=\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ find A^3 and A^{-1}, by using the Cayley-Hamilton theorem.

III. **Answer any THREE questions.** **(3x4=12)**

14. Discuss the continuity of f(x)= $\begin{cases} x^2-1, & for\ x<1 \\ 1-\frac{1}{x}, & for\ x>1 \\ 0, & for\ x=1 \end{cases}$ at x=1

15. Examine the differentiability of f(x)= $\begin{cases} x^2, x\le 3 \\ 6x-9, x>3 \end{cases}$ at x=3

16. Prove that a function which is a continuous in a closed interval is bounded.

17. Find the n^{th} derivative of $\frac{4x}{(x+1)^2(x-1)}$

18. If $y=e^{m\sin^{-1}x}$ prove that $x^2y_{n+2}-(2n+1)y_{n+1}-(n^2-m^2)y_n=0$

IV. **Answer any THREE questions.** **(3x4=12)**

19. State and prove Rolle's Theorem

20. State and prove Taylor's theorem.

21. Expand the function f(x) = log(1+x) around x=1 upto the term with x^4 by using Taylor's series.

22. Expand $e^{\sin x}$ up to the term containing x^4 by Maclaurin's expansion

23. Evaluate a) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{\log(1+x)}{x^2}\right)$ b) $\lim_{x\to 0}\left(\frac{1}{x}\right)^{2\tan x}$

V. **Answer any THREE questions.** **(3x4=12)**

24. If *u=f (r)* where $r=\sqrt{x^2+y^2+z^2}$ show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f''(r)+\frac{2}{r}f'(r)$

25. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right), x\neq y,$ show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

26. If $x=r\cos\theta, y=r\sin\theta$, then prove that $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=1.$

27. Expand $e^x siny$ by Taylor's theorem in powers of x and y as for as third degree terms.

28. Find the extreme values of the function f(x, y) =$x^3+y^3-3x-12y+2$

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