

BENGALURU CITY UNIVERSITY
I SEMESTER B.Sc. MATHEMATICS(CORE)
MODEL QUESTION PAPER - 1(2021-22 onwards) NEP

Max Marks: 60

Time: 3hrs

I. Answer any SIX questions.

(6x2=12)

1. Find the characteristic equation of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$
2. Define Rank of a matrix.
3. Find the n^{th} derivative of $\cos^2 x$.
4. Prove that every differentiable function is a continuous function.
5. If $z = x^3 - 3xy^2$ then show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.
6. Evaluate $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$
7. Verify Lagrange's mean value theorem for $f(x) = (x-1)(x-2)$ in $[0,4]$
8. Prove that there is a minimum at $(0,0)$ for the function $f(x) = x^3 + y^3 - 3xy$

II. Answer any THREE questions.

(3x4=12)

9. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$ by reducing it to row reduced echelon form.
10. Find the real value of λ for which the system the system has non-zero solution
$$\begin{aligned} (1-\lambda)x + 2y + 3z &= 0 \\ 3x + (1-\lambda)y + 2z &= 0 \\ 2x + 3y + (1-\lambda)z &= 0 \end{aligned}$$
11. Find the Eigen values and the corresponding Eigen vectors of the matrix
$$A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$$
12. State and prove Cayley-Hamilton theorem.
13. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ find A^3 and A^{-1} , by using the Cayley-Hamilton theorem.

III. Answer any THREE questions.

(3x4=12)

14. Discuss the continuity of $f(x) = \begin{cases} x^2 - 1, & \text{for } x < 1 \\ 1 - \frac{1}{x}, & \text{for } x > 1 \\ 0, & \text{for } x = 1 \end{cases}$ at $x=1$

15. Examine the differentiability of $f(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$ at $x=3$

16. Prove that a function which is continuous in a closed interval is bounded.

17. Find the n^{th} derivative of $\frac{4x}{(x+1)^2(x-1)}$

18. If $y = e^{m \sin^{-1} x}$ prove that $x^2 y_{n+2} - (2n+1)y_{n+1} - (n^2 - m^2)y_n = 0$

IV. Answer any THREE questions.

(3x4=12)

19. State and prove Rolle's Theorem

20. State and prove Taylor's theorem.

21. Expand the function $f(x) = \log(1+x)$ around $x=1$ upto the term with x^4 by using Taylor's series.

22. Expand $e^{\sin x}$ up to the term containing x^4 by Maclaurin's expansion

23. Evaluate a) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{\log(1+x)}{x^2} \right)$ b) $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{2 \tan x}$

V. Answer any THREE questions.

(3x4=12)

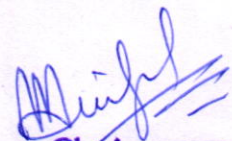
24. If $u=f(r)$ where $r = \sqrt{x^2 + y^2 + z^2}$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$

25. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$, $x \neq y$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

26. If $x = r \cos \theta$, $y = r \sin \theta$, then prove that $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} = 1$.

27. Expand $e^x \sin y$ by Taylor's theorem in powers of x and y as far as third degree terms.

28. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 2$



Chairperson
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru-560001.