BENGALURU CITY UNIVERSITY

I SEMESTER B.Sc. MATHEMATICS(CORE)

MODEL QUESTION PAPER-2 (2021-22 onwards) NEP

Time: 3hours **Max. Marks: 60**

I. Answer any SIX questions **(6x2=12)**

1. Find the value of λ for which the system of equations $7x+4y+3z=0$, $x+2y+\lambda z=0$, and $x+3y+2z=0$ has a non-trivial solution.
2. Find the eigen values of the matrix $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$
3. Find the nth derivative of $e^{2x}\sin^2 x$.
4. If $u=x^2yz$ Prove that $\frac{\partial^2 u}{\partial x \partial y}=\frac{\partial^2 u}{\partial y \partial x}$.
5. Evaluate $\lim_{x\to 0}\frac{|x|}{x}$
6. State Taylor's theorem for function of two variables.
7. Verify Rolle's theorem for the function f(x) = x^2- 6x+8 in [2,4].
8. Evaluate $\lim_{x\to 0}\frac{x-\sin x}{x^3}$.

II. Answer any THREE questions **(3x4=12)**

9. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ using elementary transformations.
10. Find λ, μ such that the system of equations $x+3y+4z=5$, $x+2y+z=3$ and $x+3y+\lambda z=\mu$ has (i) no-solution (ii) Unique solution (iii) many solutions.
11. Find the eigen values and the corresponding eigen vectors of the matrix
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$
12. Verify the Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}
13. By using Cayley-Hamilton theorem. Find the adjoint of the matrix
$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

III. Answer any THREE questions **(3x4=12)**

14. Discuss the continuity of $f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, \text{if } x\neq 2\\ 5, \text{if } x=2\end{cases}$ at x=2.

15. Discuss the differentiability of f(x) = |x| at x=0
16. Prove that a function which is a continuous in a closed interval attains its bounds in the interval.
17. State and prove Leibnitz-Theorem for finding the n^{th} derivative of product of two functions
18. Find the n^{th} derivative of (a) $\cosh 2x \cos^2 x$ (b) $\dfrac{1}{6x^2-5x+1}$.

IV. Answer any THREE questions **(3x4=12)**

19. State and prove Intermediate value theorem.
20. State and prove Lagrange's mean value theorem.
21. Using Maclaurin's expansion, prove that $\log(1+x)=1-\dfrac{x^2}{2}+\dfrac{x^3}{3}-\dfrac{x^4}{4}+\cdots$
22. Expand the function $f(x)=e^x$ around x=1 up to the term with x^5 by using Taylor's series.
23. Evaluate: $\lim\limits_{x\to 0}\dfrac{e^x+e^{-x}-2x}{x^2\sin(x)}$

V. Answer any THREE questions **(3x4=12)**

24. If $u=(x-y)^n+(y-z)^n+(z-x)^n$ prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0.$
25. State and prove Euler's theorem on homogeneous functions.
26. If $u=x+3y^2-z^3, v=2x^2-yz, w=2z^2-xy$ Evaluate $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,-1, 0).
27. Expand $2xy^2+5y-1$ in powers of (x + 1) and (y + 2) using Taylor's theorem.
28. Find the three numbers x, y, z such that $x+y+z=1$ and $xy+yz+zx$ is maximum.

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