BENGALURU CITY UNIVERSITY

I SEMESTER B.Sc. MATHEMATICS(CORE)

MODEL QUESTION PAPER - 3 (2021-22 onwards) NEP

Time: 3 hrs

Max. Marks : 60

I Answer any SIX questions. (6 x 2 = 12)

1. Find the value k in order that the matrix A = $\begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of rank 2.

2. Define eigen values and eigen vectors of a square matrix.
3. Find the nth derivative of $\log_e(2x+3)$.
4. If $z = \cos^{-1}(xy)$, find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$

5. State Lagrange's mean value theorem.
6. Name the type of discontinuity of the function

$$f(x) = \begin{cases} 3x+1, & x > 1 \\ 2x-1, & x \le 1 \end{cases} \text{ at } x=1$$

7. Show that f(x, y)=x^3+y^3–3xy+1 is minimum at the point (1,1).
8. Evaluate $\lim_{x\to 0} x^x$.

II Answer any THREE questions. (3 x 4 = 12)

9. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing it to normal form

10. Show that the equations $x+y+z=6$, $x+2y+3z=14$ and $x+4y+7z=30$ are consistent and solve them.

11. Find the eigen values and corresponding eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

12. If λ is an eigen value of a square matrix A with X as its corresponding eigen vector then Prove that (i) λ^2 is an eigen value of A^2
    (ii) $\frac{1}{\lambda}$ is an eigen value of A^{-1} provided A is non-singular.
    (iii) $\frac{1}{\lambda}|A|$ is an eigen value of adj(A) , provided A is non-singular.

13. If $A = \begin{bmatrix} 2 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then show by Cayley Hamilton theorem $A^4 = I$ and $A^3 = A^{-1}$.

III Answer any THREE questions. (3 x 4 = 12)

14. Prove that a function which continuous in a closed interval takes every value between its bounds at least once.

15. Discuss continuity of $f(x) = \frac{1}{1+e^{\left(\frac{-1}{x}\right)}}$ if $x \neq 0$ and f(0)=0 at x=0.

16. Discuss the differentiability of f(x)= $\begin{cases} 1-2x, & for\ x<0 \\ 1, & for\ 0 \le x \le 1 \\ 2x-1, & for\ x>1 \end{cases}$ at x=0

17. Find the nth derivative of $\frac{x^2-4x+1}{x^3+2x^2-x-2}$

18. Using Leibnitz rule, find the nth derivative of $(x^2+x+1)(cos2x)(cos3x)$

IV Answer any THREE questions. **(3 x 4 = 12)**

19. State and prove Cauchy's mean value theorem.

20. Expand $\tan^{-1}(x)$ in the powers of $(x - \frac{\pi}{4})$ by Taylor's theorem.

21. Expand log(1+sin x) up to the term containing x^4 using Maclaurin's series.

22. Evaluate $\lim_{x\to 0} \frac{\tan(x)-x}{x^2\tan(x)}$

23. Evaluate $\lim_{x\to 0}(1+sinx)^{cotx}$.

V Answer any THREE questions. **(3 x 4 = 12)**

24. If $z = \sin(ax+y) + \cos(ax-y)$, Prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$

25. If u = $\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

26. If $x = uvw$, $y = uv+vw+wu$ and $z = u+v+w$, Prove that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = (u-v)(v-w)(w-u)$$

27. Expand $e^x \cos y$ near the point (1, π/4) by Taylor's theorem.

28. Using the method of Lagrange's multipliers prove that the maximum volume of a rectangular box with given surface area.

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