BENGALURU CITY UNIVERSITY

I Semester B.Sc, Mathematics- Open Elective

Mathematics-I

Model Question Paper -A

Instructions: Answer all questions

Time:2Hrs. **Max. Marks:60**

Part A

I. Answer any 5 questions: **(5X3=15)**

1. Define symmetric and skew symmetric matrices.
2. Show that the system of equations $x + 2y + z = 0$, $x - 2z = 0,\ 2x + y - 3z = 0$ has only trivial solution.
3. Find the eigenvalues of the matrix $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$
4. Find the value of k such that the function $f(x) = \begin{cases} kx^2 & if\ x > 2 \\ 8 & if\ x \leq 2 \end{cases}$ is continuous at $x = 2$
5. Examine the differentiability of the function $f(x) = |x|$ at $x = 0$
6. Find the value of c by using Rolle's theorem for the function $f(x) = 8x - x^2$ in [2,6]
7. Find the area included between the parabola $y^2 = 4ax$ and its latus-rectum $x = a$
8. Write the formula for finding the surface area of the solid generated by the revolution of the curve $y = f(x)$ about the x-axis between the ordinates $x = a$ and $x = b$
9. Evaluate $\lim_{x\to 0} (cosecx - cotx)$ by using L'Hospital's Rule.

PART B

II. Answer any 3 questions: **(3X5=15)**

10. Find the rank of the matrix $A = \begin{bmatrix} 2 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing into the normal form.
11. Examine the consistency of the system of equations and solve if it is consistent, $x + 2y - z = 3,\ \ 3x - y + 2z = 1,\ \ 2x - 2y + 3z = 2$
12. Investigate for what values of λ, μ the system of equations $x + 2y + z = 8$, $2x + y + 3z = 13,\ \ 3x + 4y - \lambda z = \mu$ have (i) no solution (ii) unique solution and (iii) infinitely many solutions.
13. Find the eigenvalues and its corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

14. By using the Cayley-Hamilton's theorem find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

III. Answer any 3 questions: (3X5=15)

15. Examine the continuity of the function

$$f(x) = \begin{cases} x^2 + 3 \ for \ x > 1 \\ 2x + 2 \ for \ x \leq 1 \end{cases} \quad \text{at} \quad x = 1$$

16. Examine the differentiability of the function $f(x)$ defined by $f(x) = |x - 1|$ at $x = 1$

17. Verify the Lagrange's Mean Value theorem for the function $f(x) = x^2 - 3x + 2$ in [-2, 3]

18. Find the Taylor's series expansion of $f(x) = cosx$ about the point $x = \frac{\pi}{2}$ upto 4th degree term.

19. Evaluate: (i) $\lim_{x \to 0} \left(\frac{1-cosx}{x^2}\right)$ (ii) $\lim_{x \to 0} \left(\frac{e^x - e^{sinx}}{x - sinx}\right)$ by L'Hospital's rule.

IV. Answer any 3 questions: (3X5=15)

20. Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$.
21. Find the area bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.
22. Find the surface area of the solid generated by revolution of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x -axis
23. Find the surface area generated by revolving the curve $x = y^3$ about the y-axis from $y = 0$ to $y = 2$.

24. Show that the volume of a sphere of radius a is $\frac{4}{3}\pi a^3$

****

Chairperson
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru-560 001.