BENGALURU CITY UNIVERSITY

I Semester B.Sc, Mathematics- Open Elective

Mathematics-I

Model Question Paper -B

Instructions: Answer all questions

Time:2Hrs.   Max. Marks:60

I.   **Answer any 5 questions:**   (5 x 3 = 15)

1.   Reduce the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ to $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ using elementary transformations.

2.   Find the value of λ for which the system of equations
$$7x + 4y + 3z = 0$$
$$x + 2y + \lambda z = 0 \quad \text{has non trivial solutions.}$$
$$x + 3y + 2z = 0$$

3.   Find the eigen values of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

4.   Find the right hand limit of the function

$$f(x) = \begin{cases} 3x - 2, & when\ x \le 1 \\ 4x^2 - 3x, & when\ x > 1 \end{cases} \quad at\ \ x = 1$$

5.   Examine the differentiability of the function

$$f(x) = \begin{cases} x^2, & when\ x \le 3 \\ 6x - 9, & when\ x > 3 \end{cases} \quad at\ \ x = 3$$

6.   State Lagrange's Mean Value Theorem.

7.   Write the formula to find the length of an arc of the curve $y = f(x)$ from $x = a$ to $x = b$

8.   Find the area of the circle $x^2 + y^2 = a^2$ by using integration.

9.   Write the formula for finding the volume of solid obtained by revolving the curve $y = f(x)$ about the $x - axis$ between the lines $x = a$ and $x = b$.

II. Answer any 3 Questions: (3 x 5 = 15)

13. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 3 & 1 \\ 1 & 2 & 3 & -1 \end{bmatrix}, \text{ by reducing to the normal form.}$$

14. Find the non-trivial solutions of the system of equations

$$x + 2y + 4z = 0$$
$$x + 4y + 5z = 0$$
$$x + 2y + 7z = 0$$

15. For what values of λ and μ, the system of equations

$$x + 2y + 3z = 5$$
$$x + 3y - z = 4$$
$$x + 4y + \lambda z = \mu$$

have i) no solution ii) unique solution iii) infinitely many solutions

16. Find the eigenvalues and its corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

17. Verify Cayley-Hamilton's theorem for the matrix

$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 2 & 2 & 1 \end{bmatrix}$$

III. Answer any 3 Questions: (3 x 5 = 15)

18. Examine the continuity of the function

$$f(x) = \begin{cases} x + 1, & when\ x \geq 1 \\ x^2 + 1, & when\ x < 1 \end{cases} \quad at\ \ x = 1$$

19. Examine the differentiability of the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases} \quad at\ \ x = 0$$

20. Verify Cauchy's Mean Value theorem for the functions $f(x) = x^2$

and $g(x) = x^3$ in $[1,2]$

21. Expand the function $log_e(1 + x)$ upto the third degree term by Maclaurin's Expansion.

22. Evaluate $lim_{x \to \frac{\pi}{2}} (sin\ x)^{tan\ x}$ by L'Hospital's rule.

IV. Answer any 3 Questions: (3 x 5 = 15)

23. Find the length of the arc of the curve $y = log(sec\, x)$ from $x = 0$ to $x = \frac{\pi}{3}$.

24. Find the area of the loop of the curve ay²=x²(a-x)

25. Find the area bounded between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

26. Find the surface area of the sphere $x^2 + y^2 + z^2 = a^2$.

27. Find the volume of the solid generated by revolving the Astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis.

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