BENGALURU CITY UNIVERSITY

I Semester B.Sc, Mathematics- Open Elective

Mathematics-I

Model Question Paper - C

Instructions: Answer all questions

Time:2Hrs. **Max. Marks:60**

I. Answer any 5 questions: **(5x3=15)**

1. Find the value of 'k' such that the matrix $A = \begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of rank 2.
2. Define Eigenvalues and Eigenvectors of a square matrix.
3. Verify whether the system of equations $x + y - 2z = 5$, $x - 2y + z = -2$ and $-2x + y + z = 4$ are consistent.
4. Evaluate $\lim_{x \to 2} \left(\frac{x^2 - 3x + 2}{x - 2} \right)$
5. Discuss the Continuity of the function $f(x) = \begin{cases} x^2 - 1, & x \leq 1 \\ 1 - \frac{1}{x}, & x > 1 \end{cases}$ at $x = 1$
6. State Cauchy's Mean Value theorem.
7. Find the arc length of the catenary $y = a \cosh \left(\frac{x}{a} \right)$ from $x = 0$ to $x = a$
8. Obtain the Maclaurin's series expansion of e^x.
9. Evaluate $\lim_{x \to \frac{\pi}{2}} (secx - tanx)$ by using L'Hospital's rule.

II. Answer any 3 questions: **(3X5=15)**

10. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 6 \\ 3 & 4 & 1 & 2 \\ 5 & 6 & 3 & 2 \end{bmatrix}$ into row reduced echelon form.
11. Find the non-trivial solutions of the system

$$x + 3y - 2z = 0,\ \ 2x - y + 4z = 0\ \ and\ x - 11y + 14z = 0$$

12. For what values of λ and μ the equations
$x + y + z = 6,\ \ x + 2y + 3z = 10\ and\ x + 2y + \lambda z = \mu$

have (i) no solution (ii) unique solution (iii) infinitely many solutions.

13. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

14. Verify Cayley-Hamilton's theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

III. Answer any 3 questions: (3X5=15)

15. Examine the continuity of the function $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - e^{\frac{-1}{x}}}{e^{\frac{1}{x}} + e^{\frac{-1}{x}}}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ at $x = 0$.

16. Discuss the differentiability of the function $f(x) = \begin{cases} x^2, & x \leq 3 \\ 6x - 9, & x > 3 \end{cases}$ at $x = 3$.

17. Verify Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in [2,4].

18. Find the Taylor's series expansion for $f(x) = \sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ upto fourth degree .

19. Evaluate (i). $\lim\limits_{x \to 0} \left(\frac{a^x - b^x}{x}\right)$ (ii) $\lim\limits_{x \to 0} \left(\frac{x - \sin x}{x^3}\right)$ by using L'Hospital's rule.

IV. Answer any 3 questions: (3X5=15)

20. Find the entire length of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

21. Find the area of the loop of the curve $ay^2 = x^2(a - x)$

22. Find the surface area of the sphere of radius 'a'.

23. Find the volume of a solid generated by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $(a > b)$, about the x - axis.

24. Prove that the volume of the solid generated by revolution of the loop of the curve $2ay^2 = x(x - a)^2$ about the x-axis is $\frac{\pi a^3}{24}$.

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BENGALURU CITY UNIVERSITY

I SEMESTER B.Sc (EFFECTIVE 2021-22 ONWARDS)

QUESTION BANK

MATHEMATICS OPEN ELECTIVE

MATRICES

Questions carrying 2 marks

1. Define Symmetric , Skew-Symmetric matrices & give ~~an~~ examples.
2. Define Equivalent matrices.
3. Define Echelon form of a matrix.
4. Define Rank of a matrix.
5. Define Normal form of a matrix.
6. Define Homogeneous and non-Homogeneous system of equations.
7. What are the necessary & sufficient conditions for the system of non-homogeneous linear equations to be consistent?
8. Find the value of 'λ' for which the system of equations $2x - y + 2z = 0$, $3x + y - z = 0$ & $\lambda x - 2y + z = 0$ has a non-trivial solution.
9. Define Eigen values and Eigen vectors of a square matrix.
10. Find the Eigen values of the following matrices

    a) $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$
11. State Cayley-Hamilton's theorem.
12. Verify Cayley-Hamilton's theorem for the following matrices.

    a) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$
13. Find the inverse of the following square matrices by using Cayley-Hamilton theorem

    a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$
14. Find 'k' if the matrix $A = \begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of rank 2

Questions carrying 5 marks

1. Reduce the following matrices into Row-reduced Echelon form and find its rank.

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 6 \\ 3 & 4 & 1 & 2 \\ 5 & 6 & 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$

2. Reduce the following matrices into Normal form and find its rank

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ d) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

3. Check the consistency of the following system of equations, if consistent then solve.

a) $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30$

b) $x + y + z = -3,\ 3x + y - 2z = -2,\ 2x + 4y + 7z = 7$

c) $x + 2y - z = 3,\ 3x - y + 2z = 1,\ 2x - 2y + 3z = 2$

4. Find all the solutions of the following system of equations

a) $x + 3y - 2z = 0,\ 2x - y + 4z = 0,\ x - 11y + 14z = 0$

b) $x + 5y + 6z = 0,\ 2x + 10y + 12z = 0,\ 4x + 20y + 24z = 0$

5. Investigate for what values of λ, μ the equations

$x + y + z = 6, x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$

have i) no solution ii) unique solution iii) many solutions

6. Find the values of λ and μ the equations

$x + 3y + 4z = 5, x + 2y + z = 3,\ x + 3y + \lambda z = \mu$

have i) no solution ii)unique solution iii) many solutions

7. For what values of λ the equations

   $x + y + z = 1, x + 2y + 4z = \lambda,\ x + 4y + 10z = \lambda^2$

   have a solution and solve them completely in each case.

8. Find the real values of λ for which the system of equations

   $x + 2y + 3z = \lambda x, 3x + y + 2z = \lambda y, 2x + 3y + z = \lambda z$

   have non-trivial solution.

9. Find the Eigen values and its corresponding Eigen vectors of the following matrices.

   a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

   c) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

   b) $\begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}$

10. Find the inverse of the following square matrices using Cayley-Hamilton's theorem

   a) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

   d) $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$

DIFFERENTIAL CALCULUS

Questions carrying 2 marks

1. Define limit of a function.
2. Define continuity of a function.
3. Define differentiability of a function.
   Discuss the continuity of $f(x) = \begin{cases} 3x+1, & x > 1 \\ 2x-1, & x \leq 1 \end{cases}$ at x=1

4. Discuss the continuity of $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \frac{1}{2} \\ 0, & x = \frac{1}{2} \\ -\frac{1}{2}, & \frac{1}{2} < x < 1 \end{cases}$ at x = $\frac{1}{2}$

5. Discuss the differentiability of $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0

6. State Intermediate value theorem.
7. State Rolle's theorem.
8. State Lagrange's Mean Value theorem.
9. State Cauchy's Mean Value theorem.
10. State Taylor's theorem.

Questions carrying 5 marks

1. Evaluate

   a) $\lim_{x \to 0} \left(\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1} \right)$

   b) $\lim_{x \to 2} \left(\frac{x^2+3x+2}{x-2} \right)$

2. If $f(x) = \begin{cases} x^2, & if\ x \leq 2 \\ 8-2x, & if\ x > 2 \end{cases}$, find $\lim_{x \to 2} f(x)$

3. If $f(x) = \begin{cases} 3-x^2, & if\ x < -2 \\ x^2-5, & if\ x > -2 \end{cases}$, find $\lim_{x \to -2} f(x)$

4. Discuss the continuity of the following functions at the given points

a) $f(x) = \begin{cases} x^2 - 1, & for\ x < 1 \\ 0 \quad , & for\ x = 1 \\ 1 - \frac{1}{x}, & for\ x > 1 \end{cases}$ at $x = 1$

b) $f(x) = \begin{cases} \frac{|x|}{x}, & for\ x \neq 0 \\ 1, & for\ x = 0 \end{cases}$ at $x = 0$

c) $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x^2}}}{1 - e^{\frac{1}{x^2}}}, & for\ x \neq 0 \\ 1, & for\ x = 0 \end{cases}$ at $x = 0$

5. Discuss the differentiability of the following functions at the given points

a) $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & for\ x \neq 0 \\ 0 \quad , & for\ x = 0 \end{cases}$ at $x = 0$

b) $f(x) = \begin{cases} x^2, & for\ x \leq 3 \\ 6x - 9, & for\ x > 3 \end{cases}$ at $x = 3$

c) $f(x) = \begin{cases} 1 - 2x, & for\ x < 0 \\ 1, & for\ 0 \leq x \leq 1 \\ 2x - 1, & for\ x > 1 \end{cases}$ at $x = 0, x = 1$

d) $f(x) = \begin{cases} x^2 - 1, & for\ x \geq 1 \\ 1 - x, & for\ x < 1 \end{cases}$ at $x = 1$

e) $f(x) = \begin{cases} 1, & for\ -\infty < x < 0 \\ 1 + \sin(x), & for\ 0 \leq x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2, & for\ \frac{\pi}{2} \leq x < \infty \end{cases}$ at $x = 0$ & $x = \frac{\pi}{2}$

6. Verify Rolle's theorem for the following functions in the given interval

a) $f(x) = x^2 - 6x + 8$ in $[2,4]$

b) $f(x) = e^x \sin(x)$ in $[0, \pi]$

c) $f(x) = \sin(x) - \cos(x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

d) $f(x) = x^3 - 3x^2 - x + 3$ in $[1,3]$

7. Verify Lagrange's Mean Value Theorem for the following functions in the given interval

a) $ff(x) = \sqrt{25 - x^2}$ in $[-3,4]$

b) $f(x) = x^2 - 3x + 2$ in $[-2,3]$

c) $f(x) = \sin(x)$ in $\left[0, \frac{\pi}{2}\right]$

8. Verify Cauchy's Mean Value theorem for the following functions in the given interval.

a) $f(x) = \log(x)$ & $g(x) = \frac{1}{x}$ in $[1, e]$

b) $f(x) = x^3$ & $g(x) = x^2$ in $[1,3]$

c) $f(x) = \sin(x)$ & $g(x) = \cos(x)$ in $[\frac{-\pi}{2}, 0]$

9. Expand the following using Taylor Series

a) $f(x) = \log(\cos(x))$ about $x = \frac{\pi}{3}$ upto 4th degree

b) $f(x) = 2x^3 + 7x^2 + x - 1$ about $x = 2$ upto 4th degree

c) $f(x) = x^5 + 2x^4 - x^2 + x - 1$ in powers of $(x + 1)$ upto 4th degree

10. Expand the following functions(if possible) in an infinite series/Expand using Maclaurin's Series upto 4th Degree

a) $f(x) = \tan^{-1} x$

b) $f(x) = \tan x$

c) $f(x) = e^{\sin x}$

d) $f(x) = \cos x$

e) $f(x) = \log(\sec x)$

f) $f(x) = \log(\sin x)$

g) $f(x) = \log(1 + \sin x)$

h) $f(x) = e^{m \sin^{-1} x}$

11. Evaluate using L'Hospital's Rule

a) $\lim\limits_{x \to \infty} \left(x^{\frac{1}{x}} \right)$

b) $\lim\limits_{x \to 1} \left(x^{\frac{1}{1-x}} \right)$

c) $\lim\limits_{x \to 0} \left(x^{\frac{3}{4+\log(x)}} \right)$

d) $\lim\limits_{x \to 0} \left(\frac{\tan x - x}{x^2 \tan x} \right)$

e) $\lim\limits_{x \to 0} \left(\frac{x - \sin x}{x^3} \right)$

f) $\lim\limits_{x \to 0} (1 + \sin x)^{\cot x}$

g) $\lim\limits_{x \to 0} \left(\frac{x - \log(1+x)}{1 - \cos x} \right)$

h) $\lim\limits_{x \to 0} \left(\frac{e^x - e^{-x} - 2x}{x^2 \sin x} \right)$

i) $\lim\limits_{x \to 1} \left((2 - x)^{\tan\left(\frac{\pi x}{2}\right)} \right)$