BENGALURU CITY UNIVERSITY

FOURTH SEMESTER B.SC., MATHEMATICS (NEP) CORE

MODEL PAPER-1

Total Marks: 60 Time: $2\frac{1}{2}$ hours

I. Answer any SIX of the following: (6X2=12)

1. Form PDE by eliminating arbitrary function from $z = f(x^2 - y^2)$
2. Solve $p - x^2 = q + y^2$
3. Find the particular integral for $(D^2 - DD' - 2D'^2)z = e^{x+2y}$
4. Write the formula to find the solution of one-dimensional heat equation
5. Find Laplace transform of $t^2 + e^{5t}$
6. Find $L^{-1}\{\frac{s-1}{(s-1)^2+9}\}$
7. Write the formula for half range Fourier cosine series
8. Find the Fourier coefficient b_n for the function $f(x) = x + x^2$ in$(-\pi, \pi)$

II. Answer any THREE of the following: (3X4=12)

9. Form the partial differential equation by eliminating arbitrary function from $z = e^{ax+by} f(ax - by)$
10. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$
11. Solve $p(1 + q) = qz$
12. Solve $x^2p^2 + y^2q^2 = z^2$
13. Solve by Charpit's method $z^2(p^2 + q^2 + 1) = 1$

III. Answer any THREE of the following: (3X4=12)

14. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = sinx\ cos2y$
15. Solve $(D^2 - 2DD' - 5D - 5D' + 6)z = e^{3x-2y}$
16. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2(\frac{\partial^2 z}{\partial y^2})$ to a canonical form
17. A tightly stretched string with fixed end points x=0 and x=l is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its point a velocity $(\frac{\partial y}{\partial t})_{t=0} = 3(lx - x^2)$ find y (x, t)
18. An insulted rod of length 'l' has its ends A and B maintained at 0^0c and 100^0c respectively until steady state condition prevails. If B is suddenly reduced to 0^0c and maintained at 0^0c find the temperature at a distance x from A at time 't'.

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IV. Answer any THREE of the following: (3X4=12)

19. Find (i) $L[e^{-t}(2\ cos5t + sin2t)]$

(ii) $L[t^3 + 4t^2 - 3t + 5]$

20. Find the Laplace transform of $t^2 cosat$

21. Find $L^{-1}\left[\frac{s+5}{(s-1)(s^2+4)}\right]$

22. Using convolution theorem find $L^{-1}\left[\frac{1}{(s+1)^2(s^2+1)}\right]$

23. Express the function in terms of unit step function and hence the Laplace transform of $f(t) = \begin{cases} 1\ ; 0 \le t \le 1 \\ t\ ; 1 < t \le 2 \\ t^2\ ; t > 2 \end{cases}$

V. Answer any THREE of the following: (3X4=12)

24. Obtain the Fourier series for e^x in $(-\pi, \pi)$

25. Obtain the Fourier series of $f(x) = \frac{\pi - x}{2}$ in $0 < x < 2\pi$

26. Obtain the Fourier half range sine series of $f(x) = x^2\ in\ 0 < x < \pi$

27. Express $f(x) = \begin{cases} 1, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier Integral

28. Find Fourier cosine transform of $f(x) = \begin{cases} 1, & 0 \le x < a \\ 0, & x \ge a \end{cases}$

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FOURTH SEMESTER B.SC., MATHEMATICS (NEP) CORE

MODEL PAPER-2

Total Marks:60 Time: $2\frac{1}{2}$ hours

I. Answer any SIX of the following: (6X2=12)

1. Form the partial differential equation by eliminating arbitrary constants from $z = (x-a)^2 + (y-b)^2$
2. Solve $pq + p + q = 0$
3. Solve $(D^2 + 4DD' - 5D'^2)z = 0$
4. Classify the partial differential equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}$
5. Find $L\{\cos^2 2t\}$
6. Find $L^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\}$
7. Find the Fourier co-efficient a_n for the function $f(x) = x^2$ in $(-\pi, \pi)$
8. Write formula for Fourier sine transform

II. Answer any THREE of the following: (3X4=12)

9. Form the partial differential equation by eliminating arbitrary function from $\emptyset(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
10. Solve $(mz - ny)p + (nx - lz)q = ly - mx$
11. Solve $p + q = sinx + siny$
12. Solve $p^2 + q^2 = z^2(x + y)$
13. Find the complete integral of $z = pq$ by Charpit's method

III. Answer any THREE of the following: (3X4=12)

14. Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y}$
15. Solve $(D + D')(D + D' - 2)z = \cos(x + 2y)$
16. Reduce the equation $r + 2s + t = 0$ to canonical form
17. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the condition $u = 0$ when $x = 0 \text{ and } x = \pi$, $\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x, 0) = x \,; 0 < x < \pi$

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18. A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is

$$u(x,0) = \begin{cases} x & ; \ 0 \le x \le 50^\circ \\ 100 - x & ; 50^\circ \le x \le 100^\circ \end{cases}$$

Find the temperature $u(x,t)$ at any time $'t'$

IV. Answer any THREE of the following: (3X4=12)

19. Find i. $L(sin5t\ cos2t)$

ii. $L(e^{2t}\cos^2 t)$

20. Find $L\left(\frac{cos2t-sin3t}{t}\right)$

21. Find inverse Laplace transform of $slog\left[\frac{s+4}{s-4}\right]$

22. Verify convolution theorem for the function $f(t) = t\,; g(t) = cost$

23. If $f(t) = t^2\,, 0 < t < 2\ \ and\ f\ is\ periodic\ of\ period$ 2 then find $L\{f(t)\}$

V. Answer any THREE of the following: (3X4=12)

24. Obtain Fourier series of $f(x) = |x|\ in\ (-\pi,\pi)$

25. Obtain Fourier series of the function in $(0,2\pi)$ defined by

$$f(x) = \begin{cases} x & ; \ 0 \le x \le \pi \\ 2\pi - x & ; \ \pi \le x \le 2\pi \end{cases}$$

26. Obtain Fourier half range sine series of $f(x) = (x-1)^2$ in (0,1)

27. Find the Fourier integral expansion of

$$f(x) = e^{-ax};\ x > 0\ \ and\ \ f(-x) = f(x),\ a > 0$$

28. Find Fourier cosine transform of the function $f(x) = \begin{cases} x & ; \ 0 < x < \frac{\pi}{2} \\ 0 & ; \ \ x > \frac{\pi}{2} \end{cases}$

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FOURTH SEMESTER B.SC., MATHEMATICS (NEP) CORE
MODEL PAPER-3

Total Marks: 60 **Time: $2\frac{1}{2}$ hours**

I. Answer any SIX of the following: (6X2=12)

1. Form partial differential equation by eliminating arbitrary constants from the function $z = a^2x^2 + b^2y^2$
2. Solve $p^2 - q^2 = 1$
3. Find the complementary function for $(D^2 - 4DD' + 4D'^2)z = 0$
4. Find the particular integral of $(2D^2 - DD' - 3D'^2)z = 3e^{x-2y}$
5. Find $L\{t\ cos2t\}$
6. If $L\{f(t)\} = F(s)$ then prove that $L\{f(t)\} = sF(s) - f(0)$
7. Write the formula for fourier coefficients of the fourier series of f(x) in the interval (a, a+2l)
8. Write the formula for Fourier cosine transform

II. Answer any THREE of the following: (3X4=12)

9. Form a partial differential equation by eliminating arbitrary function from $z = f(xy + z^2, x + y + z)$
10. Solve $p\ cotx + q\ coty = cotz$
11. Solve $p^2 + q^2 = x + y$
12. Solve $z^2(p^2 + q^2 + 1) = 1$
13. Solve by Charpit's method $px + qy = pq$

III. Answer any THREE of the following: (3X4=12)

14. Solve $(D^2 - 2DD' + D'^2)z = 12xy$
15. Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$
16. Reduce $\frac{\partial^2 z}{\partial x^2} + x^2\left(\frac{\partial^2 z}{\partial y^2}\right) = 0$ to a canonical form
17. A lightly stretched string with fixed end points x=0 and x=l is initially in a position given by $y = y_0 \sin^3(\frac{\pi x}{l})$. If it is released from rest from this position, find the displacement of y(x,t).
18. Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the condition
   (i) u(0,t)=0, u(1,t)=0 for all t (ii)$u(x, 0) = x^2 - x\ ; 0 \le x \le 1$

(3X4=12)

IV. Answer any THREE of the following:

19. Find L {f(t)} if $f(t) = \begin{cases} t^2, & 0 < t < 4 \\ 10, & t > 4 \end{cases}$

20. If $L\{f(t)\} = F(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)ds.$

    Hence evaluate $L\left\{\frac{sint}{t}\right\}$

21. Find inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$

22. Using convolution theorem find inverse Laplace transform of $\frac{1}{s(s+1)^3}$

23. Express the function $f(t) = \begin{cases} 2t \ ; & 0 < t < \pi \\ 1 \ ; & t > \pi \end{cases}$ in terms of unit step function and hence find its laplace transform

V. Answer any THREE of the following

(3X4=12)

24. Obtain the Fourier series of $f(x) = |x| \ in \ (-\pi, \pi)$

25. If $f(x) = \left(\frac{\pi - x}{2}\right)^2 \ in \ 0 < x < 2\pi$. Find the Fourier series

26. Obtain the Fourier half range cosine series for the function

$$f(x) = \sin x \ in \ (0, \pi)$$

27. Show that $\int_0^\infty \frac{sin\alpha\pi \ sin\alpha x}{1-\alpha^2} d\alpha = \begin{cases} \frac{\pi}{2} sinx \ ; & -\pi \le x \le \pi \\ 0 \ ; & otherwise \end{cases}$

28. Find the Fourier sine transform of $f(x) = e^{-ax} \ ; \ a > 0$

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