

BENGALURU CITY UNIVERSITY
FOURTH SEMESTER B.SC., MATHEMATICS (NEP) CORE
MODEL PAPER-1

Total Marks: 60

Time: $2\frac{1}{2}$ hours

I. Answer any SIX of the following:

(6X2=12)

1. Form PDE by eliminating arbitrary function from $z = f(x^2 - y^2)$
2. Solve $p - x^2 = q + y^2$
3. Find the particular integral for $(D^2 - DD' - 2D'^2)z = e^{x+2y}$
4. Write the formula to find the solution of one-dimensional heat equation
5. Find Laplace transform of $t^2 + e^{5t}$
6. Find $L^{-1}\left\{\frac{s-1}{(s-1)^2+9}\right\}$
7. Write the formula for half range Fourier cosine series
8. Find the Fourier coefficient b_n for the function $f(x) = x + x^2$ in $(-\pi, \pi)$

II. Answer any THREE of the following:


(3X4=12)

9. Form the partial differential equation by eliminating arbitrary function from $z = e^{ax+by} f(ax - by)$
10. Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$
11. Solve $p(1 + q) = qz$
12. Solve $x^2p^2 + y^2q^2 = z^2$
13. Solve by Charpit's method $z^2(p^2 + q^2 + 1) = 1$

III. Answer any THREE of the following:

(3X4=12)

14. Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$
15. Solve $(D^2 - 2DD' - 5D - 5D' + 6)z = e^{3x-2y}$
16. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \left(\frac{\partial^2 z}{\partial y^2}\right)$ to a canonical form
17. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its point a velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3(lx - x^2)$ find $y(x, t)$
18. An insulated rod of length 'l' has its ends A and B maintained at 0°C and 100°C respectively until steady state condition prevails. If B is suddenly reduced to 0°C and maintained at 0°C find the temperature at a distance x from A at time 't'.


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IV. Answer any THREE of the following:

(3X4=12)

19. Find (i) $L[e^{-t}(2 \cos 5t + \sin 2t)]$

(ii) $L[t^3 + 4t^2 - 3t + 5]$

20. Find the Laplace transform of $t^2 \cos at$

21. Find $L^{-1} \left[\frac{s+5}{(s-1)(s^2+4)} \right]$

22. Using convolution theorem find $L^{-1} \left[\frac{1}{(s+1)^2(s^2+1)} \right]$

23. Express the function in terms of unit step function and hence the

$$\text{Laplace transform of } f(t) = \begin{cases} 1 & ; 0 \leq t \leq 1 \\ t & ; 1 < t \leq 2 \\ t^2 & ; t > 2 \end{cases}$$

V. Answer any THREE of the following:

(3X4=12)

24. Obtain the Fourier series for e^x in $(-\pi, \pi)$

25. Obtain the Fourier series of $f(x) = \frac{\pi-x}{2}$ in $0 < x < 2\pi$

26. Obtain the Fourier half range sine series of $f(x) = x^2$ in $0 < x < \pi$

27. Express $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ as a Fourier Integral

28. Find Fourier cosine transform of $f(x) = \begin{cases} 1, & 0 \leq x < a \\ 0, & x \geq a \end{cases}$



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MODEL PAPER-2

Total Marks:60

Time: $2\frac{1}{2}$ hours

I. Answer any SIX of the following:

(6X2=12)

1. Form the partial differential equation by eliminating arbitrary constants from $z = (x - a)^2 + (y - b)^2$
2. Solve $pq + p + q = 0$
3. Solve $(D^2 + 4DD' - 5D'^2)z = 0$
4. Classify the partial differential equation $\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} \right)$
5. Find $L\{\cos^2 2t\}$
6. Find $L^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\}$
7. Find the Fourier co-efficient a_n for the function $f(x) = x^2$ in $(-\pi, \pi)$
8. Write formula for Fourier sine transform

II. Answer any THREE of the following:

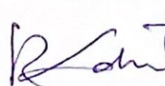
(3X4=12)

9. Form the partial differential equation by eliminating arbitrary function from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$
10. Solve $(mz - ny)p + (nx - lz)q = ly - mx$
11. Solve $p + q = \sin x + \sin y$
12. Solve $p^2 + q^2 = z^2(x + y)$
13. Find the complete integral of $z = pq$ by Charpit's method

III. Answer any THREE of the following:

(3X4=12)

14. Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x+3y}$
15. Solve $(D + D')(D + D' - 2)z = \cos(x + 2y)$
16. Reduce the equation $r + 2s + t = 0$ to canonical form
17. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ under the condition $u = 0$ when $x = 0$ and $x = \pi$, $\frac{\partial u}{\partial t} = 0$ when $t = 0$ and $u(x, 0) = x$; $0 < x < \pi$


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18. A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is

$$u(x, 0) = \begin{cases} x & ; 0 \leq x \leq 50^\circ \\ 100 - x & ; 50^\circ \leq x \leq 100^\circ \end{cases}$$

Find the temperature $u(x, t)$ at any time 't'

IV. Answer any THREE of the following:

(3X4=12)

19. Find i. $L(\sin 5t \cos 2t)$

ii. $L(e^{2t} \cos^2 t)$

20. Find $L\left(\frac{\cos 2t - \sin 3t}{t}\right)$

21. Find inverse Laplace transform of $\text{slog} \left[\frac{s+4}{s-4} \right]$

22. Verify convolution theorem for the function $f(t) = t ; g(t) = \cos t$

23. If $f(t) = t^2, 0 < t < 2$ and f is periodic of period 2 then find $L\{f(t)\}$

V. Answer any THREE of the following:

(3X4=12)

24. Obtain Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$

25. Obtain Fourier series of the function in $(0, 2\pi)$ defined by


$$f(x) = \begin{cases} x & ; 0 \leq x \leq \pi \\ 2\pi - x & ; \pi \leq x \leq 2\pi \end{cases}$$

26. Obtain Fourier half range sine series of $f(x) = (x - 1)^2$ in $(0, 1)$

27. Find the Fourier integral expansion of

$$f(x) = e^{-ax}; x > 0 \text{ and } f(-x) = f(x), a > 0$$

28. Find Fourier cosine transform of the function $f(x) = \begin{cases} x & ; 0 < x < \frac{\pi}{2} \\ 0 & ; x > \frac{\pi}{2} \end{cases}$


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Total Marks: 60

Time: $2\frac{1}{2}$ hours

(6X2=12)

I. Answer any SIX of the following:

1. Form partial differential equation by eliminating arbitrary constants from the function $z = a^2x^2 + b^2y^2$
2. Solve $p^2 - q^2 = 1$
3. Find the complementary function for $(D^2 - 4DD' + 4D'^2)z = 0$
4. Find the particular integral of $(2D^2 - DD' - 3D'^2)z = 3e^{x-2y}$
5. Find $L\{t \cos 2t\}$
6. If $L\{f(t)\} = F(s)$ then prove that $L\{f(t)\} = sF(s) - f(0)$
7. Write the formula for fourier coefficients of the fourier series of $f(x)$ in the interval $(a, a+2l)$
8. Write the formula for Fourier cosine transform

II. Answer any THREE of the following:

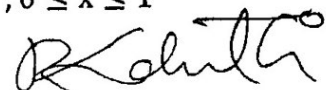
(3X4=12)

9. Form a partial differential equation by eliminating arbitrary function from $z = f(xy + z^2, x + y + z)$
10. Solve $p \cot x + q \cot y = \cot z$
11. Solve $p^2 + q^2 = x + y$
12. Solve $z^2(p^2 + q^2 + 1) = 1$
13. Solve by Charpit's method $px + qy = pq$

III. Answer any THREE of the following:

(3X4=12)

14. Solve $(D^2 - 2DD' + D'^2)z = 12xy$
15. Solve $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$
16. Reduce $\frac{\partial^2 z}{\partial x^2} + x^2 \left(\frac{\partial^2 z}{\partial y^2}\right) = 0$ to a canonical form
17. A lightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement of $y(x,t)$.
18. Solve $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ subject to the condition
 (i) $u(0,t)=0, u(1,t)=0$ for all t (ii) $u(x,0) = x^2 - x ; 0 \leq x \leq 1$



IV. Answer any THREE of the following:

(3X4=12)

19. Find $L\{f(t)\}$ if $f(t) = \begin{cases} t^2, & 0 < t < 4 \\ 10, & t > 4 \end{cases}$

20. If $L\{f(t)\} = F(s)$ then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s) ds.$

Hence evaluate $L\left\{\frac{\sin t}{t}\right\}$

21. Find inverse Laplace transform of $\frac{1}{s(s+1)(s+2)}$

22. Using convolution theorem find inverse Laplace transform of $\frac{1}{s(s+1)^3}$

23. Express the function $f(t) = \begin{cases} 2t & ; 0 < t < \pi \\ 1 & ; t > \pi \end{cases}$ in terms of unit step function and hence find its laplace transform

V. Answer any THREE of the following

(3X4=12)

24. Obtain the Fourier series of $f(x) = |x|$ in $(-\pi, \pi)$

25. If $f(x) = \left(\frac{\pi-x}{2}\right)^2$ in $0 < x < 2\pi$. Find the Fourier series

26. Obtain the Fourier half range cosine series for the function

$$f(x) = \sin x \text{ in } (0, \pi)$$

27. Show that $\int_0^\infty \frac{\sin \alpha x \sin \alpha y}{1-\alpha^2} d\alpha = \begin{cases} \frac{\pi}{2} \sin x & ; -\pi \leq x \leq \pi \\ 0 & ; \text{otherwise} \end{cases}$

28. Find the Fourier sine transform of $f(x) = e^{-ax}$; $a > 0$

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