K.L.E Society's
S. Nijalingappa College
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PG Department of Mathematics
QUESTION BANK

Magnetohydrodynamics

1. Explain :
    1. Different systems of units used in electromagnetic theory.
    2. Basic principles of Magnetohydrodynamics.
3. Derive Gauss-law for continuous distribution of charges in an element of volume in the form $\nabla.\vec{D} = \rho_e$, where the quantities have their usual meaning.
4. Show that in electrostatics the normal component of dielectric field is continuous across the interface of two media
5. Derive Ampere's law in its usual form $\nabla \times \vec{H} = \vec{J}$, where the quantities have their usual meaning
6. Derive conservation of charges equation in the form $\frac{\partial \rho_e}{\partial t} + \nabla.\vec{J} = 0$, where the quantities have their usual meaning.
7. State the law of conservation of mass and hence derive the equation for conservation of mass in the form, $\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{q}) = 0$, where the quantities have their usual meaning
8. Write a short note on boundary conditions for velocity and temperature.
9. Show that Kelvin's circulation theorem is not valid in MHD, in general.
10. Prove that the angular velocity of a perfectly conducting fluid rotating steadily about its axis of symmetry is constant along the magnetic field lines.
11. For a steady flow of a perfectly conducting fluid along the x-direction and magnetic field acting along the z-direction, show that the magnetic field is proportional to density. Here, all the flow variables are functions of x only.
12. Define a force-free magnetic field. Derive the basic equations for such a field
13. State and prove Chandrasekhar's theorem on a force free magnetic field.

14. Show that the Lorentz force can be expressed as a sum of two surface forces; one in the direction of magnetic field and the other in the direction normal to the surface.

15. Derive one-dimensional Alfven wave equations for an incompressible perfectly conducting fluid in the presence of a uniform vertical magnetic field.

16. Obtain the governing wave equations for magneto-acoustic waves in a compressible perfectly conducting non-viscous stationary fluid in the presence of a horizontal magnetic field. Show that sound waves travel faster in the presence of a magnetic field.

17. Show that the phase velocity of the above magneto acoustic waves satisfies the dispersion relation $C^4 - (a^2 + A^2)C^2 + A^2a^2 = 0$, where the quantities have their usual meaning.

18. Define hydromagnetic Couette flow. Obtain the velocity distribution for the same and discuss the effect of magnetic field.

19. Write a short note on Prandtl number.

20. (a) Write a short note on
    (i) Electrostatic and electromagnetic system of units
    (ii) Dielectric materials.

21. State and proveAmpere's law in its standard form.

22. Explain the fundamental concept behind the study of MHD and show that the Lorentz force is a retarding force.

23. Derive the magnetic induction equation for an incompressible fluid in its standard form and write a short note on magnetic Reynolds number

24. With the usual notations, derive the energy equation for an incompressible electrically conducting fluid.

25. Derive the analogue of Helmholtz vorticity equation in MHD.

26. Show that the angular velocity of a perfectly conducting fluid body rotating steadily about the axis of symmetry in a magnetic field does not vary along the magnetic field lines.

27. Show that the Lorentz force is always conservative in magnetostatics and furtherdeduce that $p + \rho\Omega = constant$, along the $\vec{J}$ and $\vec{B}$ lines, where the quantitieshave their usual meaning.

28. Explain the Bennet pinch and discuss any two instabilities associated with it.

29. Show that the Lorentz force can be expressed as a sum of two forces: one acting in the direction of the magnetic field and the other in a direction normal to the surface.

30. Explain the causes for the propagation of Alf$\acute{v}$en waves.

31. Derive the one-dimensional Alf$\acute{v}$en wave equations for an incompressible electrically conducting fluid.

32. Explain briefly the experiments conducted by Lundquist and Lehnert to show the existence of Alf$\acute{v}$en waves.

33. Obtain the velocity and temperature distributions for one-dimensional hydromagneticplane Couette flow.

34. For any graph G, prove that $K(G) \leq \lambda(G) \leq \delta(G)$, where the notations have theirusual meanings.

35. Let G be a graph of order p with $p \geq 2$ and let k be an integer such that $1 \leq k \leq p-1$. If $\deg(v) \geq \frac{p+k-2}{2}$, for every vertex v of G, then show that G is k-connected.

36. Construct the Harary graph $H_{4,6}$.

37. Prove that every non-planar graph has a subgraph which is homeomorphic to either K_5 or $K_{3,3}$.

38. Prove that every planar connected graph G contains a vertex of degree atmost 5.

39. Define crossing number. Find the crossing number of Petersen graph.

40. Define chromatic number of a graph. Find the chromatic number of

    i) Complete graph, K_p $(p \geq 1)$.

    ii)Cycle graph, C_p $(p \geq 3)$.

    iii) Complete bipartite graph, $K_{m,n}$, $1 \leq m \leq n$.

41. Prove that a graph is bicolorable if and only if it has no odd cycles.

42. If G is a graph with p-vertices, then show that $\chi(G) \geq \frac{p}{p-\delta(G)}$, where $\delta(G)$ is the minimum degree of G.

43. Prove that a map is k-face colorable if and only if its dual is k-vertex colorable.

44. State and prove five color theorem.

45. Define chromatic polynomial. Determine the chromatic polynomial of a tree with p-vertices.

46. Show that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting paths.

47. Find the number of perfect matching in the following:
    i) Complete bipartite graph, $K_{n,n}$, $n \geq 1$

    ii) Complete graph, K_{2p} , $p \geq 1$.
48. If G is k-regular bipartite graph with $k > 0$, then show that G has a perfect matching
49. Define a factor of a graph. Show that the complete graph K_{2n} is 1-factorable.
50. State and prove Havel-Hakimi theorem.
51. Check whether the degree sequence D= {4,3,3,3,2,2,2,1} is graphical or not, draw it if graphical.
52. Prove that every complete tournament has a directed Hamiltonian path.
53. Show that every acyclic digraph G has at least one vertex with zero in-degree and at least one vertex with zero out-degree
54. Show that every strong tournament with p-vertices has a cycle of length n, for $n = 3,4,\ldots,p$.
55. Define the following with an example:
    i) Domination number.

    ii) Minimal dominating set.
56. Show that every minimum dominating set is minimal but the converse is not true.
57. Show that for any graph G with p-vertices $d(G) + d(\bar{G}) = p + 1$, if and only if G is isomorphic with K_p or $\overline{K_p}$.
58. Define total domination. Prove that $\gamma(G) \leq \gamma_t(G) \leq 2\gamma(G)$, where the notations have their usual meanings.