K.L.E Society's
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PG Department of Mathematics
QUESTION BANK

Mathematical Methods

1. Using Laplace transform find the solution of $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, t > 0$ subjected to the condition $u(x,0) = 6e^{-3x}$.

2. Find the integral equation corresponding to the following differential equation with the given initial conditions:
$$\frac{d^n y}{dx^n} + a_1(x)\frac{d^{n-1}y}{dx^{n-1}} + a_2(x)\frac{d^{n-2}}{dx^{n-2}} + \ldots + a_n(x)y = \phi(x),$$
where $a_i(x)$, $(i = 1,2 \ldots n)$ are continuous functions and the initial conditions are: $y(0) = C_0, y'(0) = C_1 \ldots y^{n-1}(0) = C_{n-1}$.

3. Prove that $u(x) = \cos 2x$ is a solution of the integral equation
$u(x) = \cos x + 3\int_0^\pi K(x,t)\ u(t)\ dt$, where the kernel $K(x,t)$ is given by
$$K(x,t) = \begin{cases} \sin x \cos t, & 0 \le x \le t \\ \cos x \sin t, & t \le x \le \pi. \end{cases}$$

4. Form an integral equation corresponding to the differential equation
$y'' - 2xy' - 3y = 0;\ y(0) = 1,\ y'(0) = 0.$

5. Explain the method of separable kernels to solve the homogeneous Fredholm integral equation of the second kind $y(x) = \lambda \int_a^b K(x,t)\ y(t)\ dt$.

6. Solve the following integral equation $u(x) = 1 + \lambda \int_0^1 (x+t)\ u(t)\ dt$, by the method of successive approximation up to second order for $u_0(x) = 1$.

7. Solve the Fredholm integral equation $y(x) = x + \int_0^{1/2} y(t)\ dt$ with the help of resolvent kernel.

8. Find the resolvent kernel of the Volterra integral equation $y(x) = f(x) + \lambda \int_a^x K(x,t)\ y(t)\ dt$ with the kernel $K(x,t) = 1$.

9. Solve the integral equation $\int_0^1 \frac{y(x)}{\sqrt{t-x}} dx = 1 + t + t^2$ by the method of Laplace transform.

10. Solve the one dimensional heat equation using Laplace transform subjected to the conditions $u(x, 0) = 10 \sin 4\pi x\,,\ \ 0 \le x \le 5,\ \ u(0, t) = 0 = u(5, t),\ \ t \ge 0.$

11. Using Fourier transform find the solution of one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = 9\frac{\partial^2 u}{\partial x^2}$ subjected to the conditions: $u(0, t) = 0 = u(2, t), u(x, 0) = 0.05\,x\,(2 - x)$ and $u_t(x, 0) = 0$, where $0 < x < 2,\ \ t > 0.$

12. Define homogenous Fredholm integral equation of second kind.

Solve $y(x) = \lambda \int_0^{2\pi} \sin(x + t)\, y(t) dt.$

13. Obtain the resolvent kernel and resolve the integral equation with the following kernel $k(x, y) = e^{x-y}$

14. Explain the method of successive approximation for a nonhomogenous Volterra integral equation of a second kind.

15. Show that $\varphi(x) = \frac{3}{4}(x + 1)$ is a solution of the Volterra integral equation $\varphi(x) = x + \frac{1}{2}\int_{-1}^{1}(t - x)\varphi(t)dt$ using Neumann series method.

16. Transform integral equation corresponding to: $y''' - 2xy = 0;\ y(0) = \frac{1}{2},$ $y''(0) = y'(0) = 1.$

17. If $\varphi_m(x)$ and $\varphi_n(x)$ are fundamental functions of symmetric kernel $k(x, y)$ for the eigenvalues $\lambda_m,\ \lambda_n\ (\lambda_m \ne \lambda_n),$ then prove that $\int_a^b \varphi_m(x)\varphi_n(x)dx = 0$

18. Find the asymptotic expansion of $\int_0^x \frac{cost}{t} dt$ as $x \to \infty.$

19. Derive Laplace method for obtaining the asymptotic behavior as $x \to \infty$ of the form $\int_a^b f(t)e^{x\varphi(t)}\, dt.$

20. Use method of stationary phase to find the leading order behavior of the integral $\int_0^1 \cos(xt^4)\, \text{tan}t\, dt$ as $t \to \infty.$

21. Find the solution of the initial value problem $y'' + (1{-}\in x)y = 0$ with $y(0) = 1$ and $y'(0) = 1.$

22. By using the method of matched asymptotic find the solution of the boundaryvalue problem $\in y'' + (1{+}\in)y' + y = 0$ with $y(0) = 0, y(1) = 1.$

23. Obtain the two-term perturbation solution for Duffing equation by applying Poincare-Lindstedt method. **6**

24. Derive the Exponential WKB method.

25. Evaluate using Watson lemma: $I(x) = \int_0^{\pi/2} e^{-xsint}\, dt$

26. Solve the one dimensional wave equation using Fourier transform subject to the conditions $y(x,0) = f(x),\ y(0,t) = 0 = y(L,t)$.

27. DefineFredholm integral equation. Solve $y(x) = 1 + \int_0^1 (1 + e^{x+t} y(t))dt$.

28. Solve $\varphi(x) = 1 + \int_0^x (x-y)\varphi(y)dy$ by the method of successive approximation.

29. Determine the solution of the following integral equation $u' + 3u + 2\int_0^t u(x)dx = 2$,

$u(0) = 0$ using Laplace transform method.

30. Explain the method of Neumann series for a Kernel to obtain the solution of theFredholm integral equation of second kind.

31. Form an integral equation corresponding to the differential equation

$y''' - 2xy = 0;\ y(0) = \frac{1}{2}, y''(0) = y'(0) = 1.$

32. Define Hermitian kernel. Show that the eigenvalues of thesymmetric kernel are Find the asymptotic expansion of $\int_0^x t^{\frac{-1}{2}} e^{-t}\, dt$ as $x \to \infty$.

33. Use Laplace method to find the leading order behavior of $\int_0^1 sint\ e^{-xsinh^4 t}\, dt$ as $x \to \infty$.

34. State and prove Watson's lemma.

35. Evaluate using method of stationary phase: $\int_0^\infty e^{-xsinh^2 t}\, dt$ as $x \to \infty$.

36. Find the uniformly valid solution of the boundary value problem $\in y'' + y' = a$ with $y(0) = 0, y(1) = 1$.

37. Obtain the two-term perturbation solution for Vander Pol equation by applying Poincare-Lindstedt method.

38. Derive the WKB approximation method.

39. Show that the nonlinear Ricatti equation has the second order linear differential equation with variable co-efficient.

40. Solve the integral equation

$\int_0^{\infty} F(x) \sin xp \; dx = \begin{cases} 1, 0 \le p < 1 \\ 2, 1 \le p < 2 \\ 0, p \ge 2 \end{cases}$ by the method of Fourier sine transform.

41. Evaluate the following using Hankel transformation

(i) $H[\frac{cosax}{x}, 0]$ (ii) $H[\frac{sinax}{x}, 0]$.

42. Find the asymptotic approximation of the following integrals using Laplace method

(i) $I(x) = \int_0^{\pi/2} e^{-x \sin h^2 t} \, dt$ as $x \to \infty$.

(ii) $I(x) = \int_{-\pi/2}^{\pi/2} (t+2) e^{-x \cos t} \, dt$ as $x \to \infty$.

43. Find the leading term of the following integrals by using stationary phase method

(i)$I(x) = \int_0^{\pi/2} e^{i x \cos t} \, dt$ as $x \to \infty$.

(ii) $I(x) = \int_0^{\infty} \cos(xt^2 - t) \, dt$ as $x \to \infty$.

44. Apply regular perturbation method to solve the initial value problem
$y'' = -f(x)y; \; y(0) = 1, \; y'(0) = 1.$

45. Apply Poincare-Lindstedt method to find 2π periodic solution of
$y''(t) + \varepsilon[y^2(t) - 1]y'(t) + y(t) = 0; \; y(0) = a, \; y'(0) = 0.$

46. Find 1-term uniformly valid solution of $\varepsilon y'' + (1+x)y' + y = 0; y(0) = 1, \; y(1) = 1$ by the method of inner and outer expansions.

47. Find a leading order WKB solution of the Schrodinger equation
$\varepsilon^2 y'' = Q(x)\, y(x), Q(x) \ne 0$. Hence, find the solution of initial value problem

$\varepsilon^2 y'' = (1+x^2)^2 \; y(x); \; y(0) = 0, \; y'(0) = 1.$

48. Find the general solution of Riccati's equation $\frac{dy}{dx} + P(x)y(x) + Q(x)y^2(x) = R(x)$ if $y(x) = u(x)$ is a solution of this equation. Hence, solve $y' = y^2 + (1-2x)y + (x^2 - x + 1)$ if $y = x$ is one of the solutions.

49. Solve the following nonlinear differential equations:
(i) $yy'' - (y')^2 - a = 0$ (put: $y' = p$).

(ii) $yy'' = a[y']^2$(put : $y' = p$).