I SEMESTER B.Sc (Effective 2021-22 onwards)
OPEN ELECTIVE – QUESTION BANK
MATHEMATICS-I

MATRICES

Questions carrying 3marks

1. Define Symmetric & Skew-Symmetric matrices & give an example.
2. Define Equivalent matrices.
3. Define Echelon form of a matrix.
4. Define Rank of a matrix.
5. Define Normal form of a matrix.
6. Define Homogeneous and non-Homogeneous system of equations.
7. What are the necessary & sufficient conditions for the system of non-homogeneous linear equations to be consistent?
8. Find the value of 'λ' for which the system of equations $2x - y + 2z = 0$, $3x + y - z = 0$ & $\lambda x - 2y + z = 0$ has a non-trivial solution.

9. Define Eigen values and Eigen vectors of a square matrix.
10. Find the Eigen values of the following matrices

    a) $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 5 & -1 \\ 4 & 9 \end{bmatrix}$
11. State Cayley-Hamilton's theorem.
12. Verify Cayley-Hamilton's theorem for the following matrices.

    a) $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$
13. Find the inverse of the following square matrices by using Cayley-Hamilton theorem

    a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

14. Find 'k' if the matrix $A = \begin{bmatrix} 6 & k & -1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$ is of rank 2

Questions carrying 5 marks

1. Reduce the following matrices into Row-reduced Echelon form and find its rank.

a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 6 \\ 3 & 4 & 1 & 2 \\ 5 & 6 & 3 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ 3 & 2 & 6 & 7 \end{bmatrix}$

2. Reduce the following matrices into Normal form and find its rank

a) $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

3. Check the consistency of the following system of equations, if consistent then solve.
   a) $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30$
   b) $x + y + z = -3,\ 3x + y - 2z = -2,\ 2x + 4y + 7z = 7$
   c) $x + 2y - z = 3,\ 3x - y + 2z = 1,\ 2x - 2y + 3z = 2$
4. Find all the solutions of the following system of equations
   a) $x + 3y - 2z = 0,\ 2x - y + 4z = 0,\ x - 11y + 14z = 0$
   b) $x + 5y + 6z = 0,\ 2x + 10y + 12z = 0,\ 4x + 20y + 24z = 0$

5. Investigate for what values of λ, μ the equations
   $x + y + z = 6, x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$
   have i) no solution ii) unique solution iii) many solutions

6. Find the values of λ and μ the equations
   $x + 3y + 4z = 5, x + 2y + z = 3,\ x + 3y + \lambda z = \mu$
   have i) no solution ii) unique solution iii) many solutions

7. For what values of λ the equations
   $x + y + z = 1, x + 2y + 4z = \lambda,\ x + 4y + 10z = \lambda^2$

   have a solution and solve them completely in each case.

8. Find the real values of λ for which the system of equations

$$x + 2y + 3z = \lambda x,\ 3x + y + 2z = \lambda y,\ 2x + 3y + z = \lambda z$$

have non-trivial solution.

9. Find the Eigen values and its corresponding Eigen vectors of the following matrices.

a) $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix}$

d) $\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}$

10. Find the inverse of the following square matrices using Cayley-Hamilton's theorem

a) $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$

DIFFERENTIAL CALCULUS

Questions carrying 3 marks

1. Define limit of a function.
2. Define continuity of a function.
3. Define differentiability of a function.
4. Discuss the continuity of $f(x) = \begin{cases} 3x + 1, & x > 1 \\ 2x - 1, & x \le 1 \end{cases}$ at x=1
5. Discuss the continuity of $f(x) = \begin{cases} \frac{1}{2}, & 0 < x < \frac{1}{2} \\ 0, & x = \frac{1}{2} \\ -\frac{1}{2}, & \frac{1}{2} < x < 1 \end{cases}$ at x = $\frac{1}{2}$
6. Discuss the differentiability of $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0
7. State Intermediate value theorem.

8. State Rolle's theorem.
9. State Lagrange's Mean Value theorem.
10. State Cauchy's Mean Value theorem.
11. State Taylor's theorem.

Questions carrying 5 marks

1. Evaluate

   a) $\lim_{x\to 0}\left(\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}\right)$

   b) $\lim_{x\to 2}\left(\frac{x^2-3x+2}{x-2}\right)$

2. If $f(x)=\begin{cases} x^2, & if\ x\le 2\\ 8-2x, & if\ x>2\end{cases}$, find $\lim_{x\to 2} f(x)$

3. If $f(x)=\begin{cases} 3-x^2, & if\ x<-2\\ x^2-5, & if\ x>-2\end{cases}$, find $\lim_{x\to -2} f(x)$

4. Discuss the continuity of the following functions at the given points

   a) $f(x)=\begin{cases} x^2-1, & for\ x<1\\ 0, & for\ x=1\\ 1-\frac{1}{x}, & for\ x>1\end{cases}$ at $x=1$

   b) $f(x)=\begin{cases} \frac{|x|}{x}, & for\ x\ne 0\\ 1, & for\ x=0\end{cases}$ at $x=0$

   c) $f(x)=\begin{cases} \frac{e^{\frac{1}{x^2}}}{1-e^{\frac{1}{x^2}}}, & for\ x\ne 0\\ 1, & for\ x=0\end{cases}$ at $x=0$

5. Discuss the differentiability of the following functions at the given points

   a) $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & for\ x\ne 0\\ 0, & for\ x=0\end{cases}$ at $x=0$

   b) $f(x)=\begin{cases} 1-2x, & for\ x<0\\ 1, & for\ 0\le x\le 1\\ 2x-1, & for\ x>1\end{cases}$ at $x=0, x=1$

   c) $f(x)=\begin{cases} x^2-1, & for\ x\ge 1\\ 1-x, & for\ x<1\end{cases}$ at $x=1$

d) $f(x) = \begin{cases} 1, & for -\infty < x < 0 \\ 1+\sin(x), & for\ 0 \le x < \frac{\pi}{2} \\ 2+\left(x-\frac{\pi}{2}\right)^2, & for\ \frac{\pi}{2} \le x < \infty \end{cases}$ at $x = 0$ & $x = \frac{\pi}{2}$

6. Verify Rolle's theorem for the following functions in the given interval
   a) $f(x) = x^2 - 6x + 8$ in $[2,4]$
   b) $f(x) = e^x \sin(x)$ in $[0,\pi]$
   c) $f(x) = \sin(x) - \cos(x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$
   d) $f(x) = x^3 - 3x^2 - x + 3$ in $[1,3]$

7. Verify Lagrange's Mean Value Theorem for the following functions in the given interval
   a) $f(x) = \sqrt{25 - x^2}$ in $[-3,4]$
   b) $f(x) = x^2 - 3x + 2$ in $[-2,3]$
   c) $f(x) = \sin(x)$ in $\left[0, \frac{\pi}{2}\right]$

8. Verify Cauchy's Mean Value theorem for the following functions in the given interval.

a) $f(x) = \log(x)$ & $g(x) = \frac{1}{x}$ in $[1,e]$

b) $f(x) = x^3$ & $g(x) = x^2$ in $[1,3]$

9. Expand the following using Taylor Series

a) $f(x) = \log(\cos(x))$ about $x = \frac{\pi}{3}$ upto 4^{th} degree

b) $f(x) = 2x^3 + 7x^2 + x - 1$ about $x = 2$ upto 4^{th} degree

c) $f(x) = x^5 + 2x^4 - x^2 + x - 1$ in powers of $(x+1)$ upto 4^{th} degree

10. Expand the following functions(if possible) in an infinite series/Expand using Maclaurin's Series upto 4^{th} Degree
    a) $f(x) = \tan x$
    b) $f(x) = e^{\sin x}$
    c) $f(x) = \cos x$
    d) $f(x) = \log(\sec x)$
    e) $f(x) = \log(1 + \sin x)$

f) $f(x) = e^{m\sin^{-1}x}$

11. Evaluate using L'Hospital's Rule

a) $\lim_{x\to\infty}\left(x^{\frac{1}{x}}\right)$

b) $\lim_{x\to 0}\left(\frac{\tan x - x}{x^2\tan x}\right)$

c) $\lim_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$

d) $\lim_{x\to 0}(1+\sin x)^{\cot x}$

e) $\lim_{x\to 0}\left(\frac{x-\log(1+x)}{1-\cos x}\right)$

f) $\lim_{x\to 0}\left(\frac{e^x-e^{-x}-2x}{x^2\sin x}\right)$

g) $\lim_{x\to 1}\left((2-x)^{\tan\left(\frac{\pi x}{2}\right)}\right)$

INTEGRAL CALCULUS

Questions carrying 3 marks

1. Write the formula to find the length of an arc of the curve $y = f(x)$ from $x = a\ to\ x = b$
2. Find the area bounded by the x-axis & the curve $y = c\ cosh\left(\frac{x}{c}\right)$ between $x = 0$ & $x = c$
3. Find the area included between the parabola $y^2 = 4ax$ & its Latus Rectum $x = a$
4. Write the formula to find the area enclosed by the curve $y = f(x)$, the x-axis between $x = a$ & $x = b$
5. Write the formula to find the area enclosed by the curve $x = f(y)$, the y-axis between $y = c$ & $y = d$
6. Write the formula to find the surface area of the solid generated by revolving the area bounded by the curve $y = f(x)$, the x-axis & $x = a, x = b$
7. Write the formula to find the volume of revolution of the curve $y = f(x)$, between $x = a$ & $x = b$ about x-axis.
8. Find length of an arc of parabola y^2=4ax cutting off by the latus rectum

Questions carrying 5 marks

1. Find entire length of astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$
2. Prove that the volume of the solid generated by revolution of the loop of the curve $2ay^2 = x(x-a)^2$ about the x-axis is $\frac{\pi a^3}{24}$.

3. Find the length of the loop of the curve $3ay^2=x(x-a)^2$
4. Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$.

5. Find the area bounded by the curve $a^2y = x^2(x+a)$ and the x-axis
6. Find the length of the curve $4y^2 = x^3$ between $x = 0$ and $x = 1$.
7. Find the area bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.
8. Find the surface area of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ revolving about x-axis
9. Find the surface area generated by revolving the curve $x = y^3$ about the y-axis from $y = 0$ to $y = 2$.
10. Find the surface area generated by revolving the curve $x = y^3$ about the y-axis from $y = 0$ to $y = 2$.

11. Find the volume of the sphere $x^2+y^2+z^2=a^2$
12. Find the volume of the solid obtained by revolving the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about x-axis
13. Show that the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis is

    48 π