

BANGALORE CITY UNIVERSITY

V SEMESTER B.Sc MATHEMATICS-5.1

Real Analysis-I & Complex Analysis
MODEL PAPER -1

Time: $2 \frac{1}{2}$ hrs

Max marks: 60

I. Answer any SIX questions

2X6=12

1. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
2. Define Improper integral of first kind. Give an example
3. Find the locus of the point z satisfying $|z-1| \geq 2$
4. State Cauchy Riemann equations in cartesian form for a complex valued function $f(z)$ to be analytic
5. Define orthogonal surfaces.
6. State Cauchy's integral theorem
7. State Liouville's theorem
8. Define bilinear transformation

II. Answer any THREE questions

3X4=12

6. Obtain the relation between beta and gamma function

7. Prove that $\int_0^{\infty} \sqrt{y} e^{-y^2} dy \cdot \int_0^{\infty} \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$

8. State and prove Duplication formula

9. Prove that $\beta(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$

10. Prove that $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2-\sin^2 \theta}} d\theta = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$

III. Answer any THREE questions

3X4=12

11. State and prove C-R equations in polar form

12. Show that $f(z) = \log z$ is analytic and prove that $f'(z) = \frac{1}{z}$

13. If $f(z)$ is a regular function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$



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14. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.
 15. Find the analytic function $f(z) = u + iv$ where $v(x, y) = x \sin x \sin hy - y \cos x \cos hy$

IV Answer any THREE questions

3X4=12

16. Evaluate $\int_c z^2 dz$ along the straight line from $z = 0$ to $z = 3+i$
 17. If c is the circle with centre a and radius r , show that
 (i) $\int_c \frac{dz}{z-a} = 2\pi a$ (ii) $\int_c (z-a)^n dz = 0$ if $n \neq 1$
 18. State and prove Cauchy's Integral formula for a complex function
 19. Evaluate $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ where $c: |z| = 3$
 20. Evaluate $\int_c \frac{z^2 + 5}{(z-2)(z-3)} dz$ where $c: |z| = 4$

V Answer any THREE questions

3X4=12

21. Show that the transformation of $w = e^z$ maps the straight line parallel to co-ordinate axis in z -plane to orthogonal trajectories in w -plane
 22. Show that the transformation $w = \frac{1}{z}$ transforms a circle to a circle or straight line
 23. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z -plane.
 Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$
 24. Find the bilinear transformation which maps $z = 1, i, -1$ into $w = i, 0, -i$
 25. Find the fixed points of bilinear transformation $w = \frac{z}{2-z}$



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V SEMESTER B.Sc MATHEMATICS-5.1

Time: $2\frac{1}{2}$ hrs.

Real Analysis II & Complex Analysis
MODEL PAPER -2

Max marks: 60

I Answer any SIX questions

2X6=12

1. Write the duplication formula
2. Define Improper integral of second kind. Give an example.
3. Evaluate $\lim_{z \rightarrow -2i} \frac{(2z+3)(z-1)}{z^2-2z+4}$
4. Define differentiability of complex valued function at a point $z = z_0$
5. Define a harmonic function
6. State fundamental theorem of algebra
7. Write Cauchy's generalized integral formula
8. Write the cross ratio of 4 points in a bilinear transformation

II. Answer any THREE questions

3X4=12

6. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
7. Prove that $\frac{\beta(m+2, n-2)}{m(m+1)} = \frac{\beta(m, n)}{(n-1)(n-2)}$
8. Prove that $\int_0^{\infty} x e^{-x^2} dx \cdot \int_0^{\infty} x^2 e^{-x^2} dx = \frac{\pi}{16\sqrt{2}}$
9. Prove that $\int_a^{\infty} e^{2ax-x^2} dx = \frac{\sqrt{\pi}}{2} e^{a^2}$
10. Prove that $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin \theta}} d\theta \cdot \int_0^{\frac{\pi}{2}} \sqrt{\sin \theta} d\theta = \pi$

IV. Answer any THREE questions

3X4=12

11. State and prove C-R equations in Cartesian form
12. By using the definition of $f(z)$ find the derivative if $f(z) = \frac{1+z}{1-z}$
13. If $f(z) = u + iv$ is analytic prove that $\left(\frac{\partial}{\partial x} |f(z)|\right)^2 + \left(\frac{\partial}{\partial y} |f(z)|\right)^2 = |f'(z)|^2$


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14. Prove that the functions $u(x,y)$ and $v(x,y)$ are harmonic conjugates to each other iff they are constant functions
15. If $u = e^x(x \cos y - y \sin y)$ Find the analytic function $f(z)$.

IV Answer any THREE questions

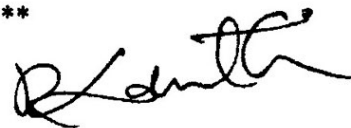
3X4=12

16. Evaluate $\int_c |z|^2 dz$ where c is the square with vertices $(0,0), (1,0), (1,1)$ and $(0,1)$
17. Evaluate $\int_{1-i}^{2+i} (2x + iy + 1) dz$ along the point $x = t + 1, y = 2t^2 - 1$
18. State and prove Cauchy's Integral theorem for complex functions
19. Evaluate $\int_c \frac{e^{2z}}{z + i\pi} dz$ along $|z - 1| = 1$
20. Evaluate $\int_c \frac{z}{(z^2 + 1)(z^2 - 9)} dz$ where $c : |z| = 2$

V Answer any THREE questions

3X4=12

21. Show that the transformation $w = \sin z$ transforms the lines parallel to real axis in z -plane into the system of ellipse in w -plane
22. Show that the transformation $w = z^2$ transforms the lines parallel to imaginary axis to set of confocal parabolas in w -plane
23. Prove that a bilinear transformation transforms circles into circles or straight lines
24. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$
25. Find the fixed points of bilinear transformation $w = \frac{3z-4}{z}$



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Real Analysis - II + Complex Analysis
MODEL PAPER -3

Time: 2 $\frac{1}{2}$ hrs

Max marks: 60

I. Answer any SIX questions

2X6=12

1. Find the value of $\int_0^{\frac{\pi}{2}} \sin^{1/2} x \cdot \cos^{3/2} x \, dx$
2. Evaluate $\int_0^{\infty} \sqrt{x} e^{-x} \, dx$
3. State Cauchy Riemann equations in polar form for a complex valued function $f(z)$ to be analytic
4. Evaluate $\lim_{z \rightarrow -i} \frac{z^2 + 1}{z^6 + 1}$
5. Define continuity of a complex function $f(z)$ at $z = z_0$
6. State Cauchy's integral formula
7. State fundamental theorem of algebra
8. Define invariant point of complex function
9. Define conformal transformation between z, w planes

II. Answer any THREE questions

3X4=12

6. Prove that $\beta(m, n) \cdot \beta(m+n, p) = \beta(n, p) \cdot \beta(m, m+p)$
7. Prove that $\int_0^1 x^m \left(\log \left(\frac{1}{x} \right) \right)^n \, dx = \frac{n!}{(m+1)^{n+1}}$, n is a positive integer
8. Show that $\int_0^{\frac{\pi}{2}} \sin^p \theta \, d\theta \cdot \int_0^{\frac{\pi}{2}} \sin^{p+1} \theta \, d\theta = \frac{\pi}{2(p+1)}$
9. Prove that $\beta(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} \, dx$
10. Prove that $\int_{-1}^1 (1+x)^{p-1} (1-x)^{q-1} \, dx = 2^{p+q-1} \beta(p, q)$

III. Answer any THREE questions

3X4=12

11. Show that $f(z) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & \text{for } z \neq 0 \\ 1 & \text{for } z = 0 \end{cases}$ is discontinuous at the origin



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12. State and prove C-R equations in cartesian form
13. Prove that $f(z) = e^{-x}(x\sin y - y\cos y)$ is harmonic and find its harmonic conjugate
14. If $u - v = (x - y)(x^2 + 4xy + y^2)$. Find $f(z)$ in terms of z
15. Find the analytic function whose real part is $\left(r + \frac{1}{r}\right)\cos\theta$

IV Answer any THREE questions

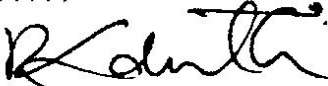
3X4=12

16. State and prove Cauchy's Integral theorem for complex functions
17. Evaluate $\int_c z^2 dz$ along the curve made up of two line segments $z = 0$ to $z = 3$ and $z = 3$ to $z = 3 + i$
17. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$
18. State and prove Liouville's theorem
19. Evaluate $\int_c \frac{e^{2z}}{(z+1)^2(z-2)} dz$ along $|z| = 3$
20. Evaluate $\int_c \frac{e^z}{z+1} dz$ where $c: |z| = 2$

V Answer any THREE questions

3X4=12

21. Show that the transformation of $w = z^2$ transforms the circle $|z-a|=r$ onto a limaçon or a cardioid
22. Find the region in the w -plane corresponding to the rectangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ in z -plane under the transformation $w = e^{\frac{i\pi}{4}z}$
23. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z -plane. Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$
24. Find the bilinear transformation which maps $z = 1, i, -1$ into $w = 2, i, -2$
25. Find the fixed points of bilinear transformation $w = \frac{z}{2-z}$


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