BANGALORE CITY UNIVERSITY

V SEMESTER B.Sc MATHEMATICS-5.1

Real Analysis-II & Complex Analysis

Time: $2\frac{1}{2}$ hrs **MODEL PAPER -1** **Max marks: 60**

I. Answer any SIX questions **2X6=12**

1. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
2. Define Improper integral of first kind. Give an example
3. Find the locus of the point z satisfying $|z-1| \geq 2$
4. State Cauchy Riemann equations in cartesian form for a complex valued function $f(z)$ to be analytic
5. Define orthogonal surfaces.
6. State Cauchy's integral theorem
7. State Liouville's theorem
8. Define bilinear transformation

II. Answer any THREE questions **3X4=12**

6. Obtain the relation between beta and gamma function
7. Prove that $\int_0^\infty \sqrt{y}\, e^{-y^2} dy \cdot \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$
8. State and prove Duplication formula
9. Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$
10. Prove that $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2-\sin^2\theta}} d\theta = \frac{\sqrt{\pi}}{4} \frac{\Gamma\left(\frac{1}{4}\right)}{\Gamma\left(\frac{3}{4}\right)}$

III. Answer any THREE questions **3X4=12**

11. State and prove C-R equations in polar form
12. Show that $f(z) = \log z$ is analytic and prove that $f'(z) = \frac{1}{z}$
13. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4\,|f'(z)|^2$

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

14. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.

15. Find the analytic function $f(z) = u + iv$ where $v(x, y) = x \sin x . \sin hy - y \cos x \cos hy$

IV Answer any THREE questions **3X4=12**

16. Evaluate $\int_c z^2 \, dz$ along the straight line from $z = 0$ to $z = 3+i$

17. If c is the circle with centre a and radius r, show that

$$(i) \int_c \frac{dz}{z-a} = 2\pi a \quad (ii) \int_c (z-a)^n \, dz = 0 \ \text{if} \ n \neq 1$$

18. State and prove Cauchy's Integral formula for a complex function

19. Evaluate $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ *where* $c : |z| = 3$

20. Evaluate $\int_c \frac{z^2 + 5}{(z-2)(z-3)} dz$ *where* $c : |z| = 4$

V Answer any THREE questions **3X4=12**

21. Show that the transformation of $w = e^z$ maps the straight line parallel to co-ordinate axis in z-plane to orthogonal trajectories in w-plane

22 .Show that the transformation $w = \frac{1}{z}$ transforms a circle to a circle or straight line

23. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z-plane. Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$

24. Find the bilinear transformation which maps $z = 1, i, -1$ *into* $w = i, 0, -i$

25. Find the fixed points of bilinear transformation $w = \frac{z}{2-z}$

****

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

BANGALORE CITY UNIVERSITY

V SEMESTER B.Sc MATHEMATICS-5.1 Real Analysis II & Complex Analysis I

Time: 2 $\frac{1}{2}$ hrs. **MODEL PAPER -2** **Max marks: 60**

I Answer any SIX questions **2X6=12**

1. Write the duplication formula
2. Define Improper integral of second kind. Give an example.
3. Evaluate $\lim_{z\to -2i} \dfrac{(2z+3)(z-1)}{z^2-2z+4}$
4. Define differentiability of complex valued function at a point $z = z_0$
5. Define a harmonic function
6. State fundamental theorem of algebra
7. Write Cauchy's generalized integral formula
8. Write the cross ratio of 4 points in a bilinear transformation

II. Answer any THREE questions **3X4=12**

6. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
7. Prove that $\dfrac{\beta(m+2,n-2)}{m(m+1)} = \dfrac{\beta(m,n)}{(n-1)(n-2)}$
8. Prove that $\int_0^\infty xe^{-x^4}dx.\ \int_0^\infty x^2e^{-x^4}dx = \dfrac{\pi}{16\sqrt{2}}$
9. Prove that $\int_a^\infty e^{2ax-x^2}dx = \dfrac{\sqrt{\pi}}{2}e^{a^2}$
10. Prove that $\int_0^{\frac{\pi}{2}} \dfrac{1}{\sqrt{\sin\theta}}d\theta.\ \int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}d\theta. = \pi$

IV. Answer any THREE questions **3X4=12**

11. State and prove C-R equations in Cartesian form
12. By using the definition of $f(z)$ find the derivative if $f(z) = \dfrac{1+z}{1-z}$
13. If $f(z) = u + iv$ is analytic prove that $\left(\dfrac{\partial}{\partial x}|f(z)|\right)^2 + \left(\dfrac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

14. Prove that the functions $u(x,y)$ and $v(x,y)$ are harmonic conjugates to each other iff they are constant functions
15. If $u = e^x(x\cos y - y\sin y)$ Find the analytic function $f(z)$.

IV Answer any THREE questions **3X4=12**

16. Evaluate $\int_c |z|^2 dz$ where c is the square with vertices (0,0),(1,0),(1,1) and (0,1)
17. Evaluate $\int_{1-i}^{2+i}(2x + iy + 1)dz$ along the point $x = t + 1, y = 2t^2$-1
18. State and prove Cauchy's Integral theorem for complex functions
19. Evaluate $\int_c \frac{e^{2z}}{z+i\pi}dz$ *along* $|z-1|=1$
20. Evaluate $\int_c \frac{z}{(z^2+1)(z^2-9)}dz$ *where* $c:|z|=2$

V Answer any THREE questions **3X4=12**

21. Show that the transformation $w = sinz$ transforms the lines parellel to real axis in z-plane into the system of ellipse in w-plane
22. Show that the transformation $w = z^2$ transforms the lines parallel to imaginary axis to set of confocal parabolas in w-plane
23. Prove that a bilinear transformation transforms circles into circles or straight lines
24. Find the bilinear transformation which maps $z = \infty, i, 0$ *into* $w = -1, -i, 1$
25. Find the fixed points of bilinear transformation $w = \frac{3z-4}{z}$

****

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

BANGALORE CITY UNIVERSITY

V SEMESTER B.Sc MATHEMATICS-5.1

Real Analysis-II & Complex Analysis

Time: 2 $\frac{1}{2}$ hrs **MODEL PAPER -3** **Max marks: 60**

I. Answer any SIX questions **2X6=12**

1. Find the value of $\int_0^{\frac{\pi}{2}} sin^{1/2}x.cos^{3/2}x\,dx$
2. Evaluate $\int_0^{\infty} \sqrt{x}\,e^{-x}dx$
3. State Cauchy Riemann equations in polar form for a complex valued function $f(z)$ to be analytic
4. Evaluate $\lim_{z\to -i} \frac{z^2+1}{z^6+1}$
5. Define continuity of a complex function $f(z)$ *at* $z = z_0$
6. State Cauchy's integral formula
7. State fundamental theorem of algebra
8. Define invariant point of complex function
9. Define conformal transformation between z,w planes

II. Answer any THREE questions **3X4=12**

6. Prove that $\beta(m,n).\beta(m+n,p) = \beta(n,p).\beta(n+p,m)$
7. Prove that $\int_0^1 x^m \left(\log\left(\frac{1}{x}\right)\right)^m dx = \frac{n!}{(m+1)^{n+1}}$, n *is a positive* integer
8. Show that $\int_0^{\frac{\pi}{2}} \sin^p\theta\,d\theta.\int_0^{\frac{\pi}{2}} \sin^{p+1}\theta\,d\theta = \frac{\pi}{2(p+1)}$
9. Prove that $\beta(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$
10. Prove that $\int_{-1}^{1} (1+x)^{p-1}(1+x)^{q-1}\,dx = 2^{p+q-1}\beta(p,q)$

III. Answer any THREE questions **3X4=12**

11. Show that $f(z) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & for\ z \neq 0 \\ 1 & for\ z = 0 \end{cases}$ is discontinuous at the origin

Chairman
Department of Mathematics
Central College Campus
Bengaluru-560 001

12. State and prove C-R equations in cartesian form

13. Prove that $f(z) = e^{-x}(xsiny\text{-}ycosy)$ is harmonic and find its harmonic conjugate

14. If $u - v = (x - y)(x^2 + 4xy + y^2)$ .Find $f(z)$ in terms of z

15. Find the analytic function whose real part is $\left(r + \frac{1}{r}\right) cos\theta$

IV Answer any THREE questions **3X4=12**

16. State and prove Cauchy's Integral theorem for complex functions

17. Evaluate $\int_c z^2 dz$ along the curve made up of two line segments $z = 0\ to\ z = 3$ and $z = 3\ to\ z = 3 + i$

17. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x^2$

18. State and prove Liouville's theorem

19. Evaluate $\int_c \frac{e^{2z}}{(z+1)^2(z-2)} dz\ \ along\ |z| = 3$

20. Evaluate $\int_c \frac{e^z}{z+1} dz\ \ where\ \ c: |z| = 2$

V Answer any THREE questions **3X4=12**

21. Show that the transformation of $w = z^2$ transforms the circle |z-a|=r onto a limacon or a cardiod

22 . Find the region in the w-plane corresponding to the rectangular region bounded by the lines $x = 0, y = 0\ and\ x + y = 1$ in z-plane under the transformation $w = e^{\frac{i\pi}{4}z}$

23. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z-plane. Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$

24. Find the bilinear transformation which maps $z = 1, i, -1\ into\ w = 2, i, -2$

25. Find the fixed points of bilinear transformation $w = \frac{z}{2-z}$

****

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001