BENGALURU CITY UNIVERSITY

V Semester B.Sc Degree Examination

Mathematics (Core) - MATDSCT 5.2

Vector Calculus and Analytical Geometry

Model Paper – 1

Time: 2 Hours and 30 Minutes **Max. Marks: 60**

I Answer any six questions **6 × 2 = 12**

1. If $\vec{r} = t\hat{i} - t^3\hat{j} + \sin t\,\hat{k}$, *find* $\frac{d\vec{r}}{dt}, \left|\frac{d\vec{r}}{dt}\right|$ *at* $t = 0$
2. Define curvature and torsion of a space curve
3. Find the unit normal vector to the surface $x^2 - y^2 + z = 2$ at (1,1,1)
4. Show that the vector $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.
5. Evaluate $\int_c 5xy\,dx + y^2dy$ where c is the curve $y = 2x^2$ from (0,0)to (1,2).
6. State the Stokes theorem.
7. Find the distance of the point (2, 3, 4) from the plane 3x-6y+2z+11=0.
8. Find the equation of the sphere whose center at (2,-3,4) and radius is 5 units.

II Answer any three questions **3 × 4 = 12**

9. If $\vec{u} = t\hat{i} + 2t^2\hat{k}$, $\vec{v} = t^3\hat{j} + t\hat{k}, \vec{w} = \hat{i} + t\hat{j} + t^2\hat{k}$ *Find* $\frac{d}{dt}\left(\vec{u} \times \left(\vec{v} \times \vec{w}\right)\right)$
10. State and prove Serenet-Frenet formula.
11. For the space curve $x = ae^u, y = ae^{-u}, z = \sqrt{2}\,au$ *ST* $\kappa = -\tau$
12. Show that the cylindrical coordinate system is orthogonal curvilinear coordinate system.
13. Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar coordinates.

III Answer any three questions **3 × 4 = 12**

14. Prove that the surfaces $4x^2y + z^3 = 4$ *and* $5x^2 - 2yz = 9x$ intersect orthogonally at the point (1,-1,2).
15. If $\vec{f} = 2x\hat{i} + 3y\hat{j} + 4z\hat{k}$ and $\phi = xy^2z^3$ find $\vec{f}.\nabla\phi$ and $\nabla|\vec{f}|^2$.
16. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ where $r^2 = x^2 + y^2 + z^2$
17. Find the constants a,b,c so that the vector $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ Is irrotational

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18. If ϕ and $\vec{F}$ are scalar point, vector point functions respectively then, prove that

$$curl(\phi\vec{F}) = \phi curl\vec{F} + (grad\phi)\times\vec{F}$$

IV Answer any three questions $3 \times 4 = 12$

19. Evaluate $\iint_R xy(x+y)dxdy$, where R is the region bounded between the parabola $y = x^2$ *and the line* $y = x$.
20. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using double Integration.
21. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} xyz\, dxdydz$.
22. Find the volume of the Tetrahedron bounded by the plane $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
23. Evaluate using Gauss divergence theorem $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ taken over the rectangular parelopiped $x = 0, y = 0, z = 0$ and $x = a, y = b, z = c$.

V Answer any three questions $3 \times 4 = 12$

24. Find the distance between the parallel planes 2x-y+3z+4=0 and 6x-3y+9z-3=0.
25. Find the image of the point (-3,0,1) in the plane 4x-3y+2z=19.
26. Find the distance between the lines $\frac{x-2}{3} = \frac{y+1}{0} = \frac{z-3}{-1}$ and $\frac{x+1}{3} = \frac{y-2}{0} = \frac{z+4}{-1}$.
27. Find the equation of the sphere which passes through the points (1,0,0), (0,1,0), (0,0,1) and whose center lies on the plane 3x-y+z=2.
28. Find the equation of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane 2x+2y-z=0.

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Mathematics (Core) - MATDSCT 5.2

Vector Calculus and Analytical Geometry

Model Paper – 2

Time: 2 Hours and 30 Minutes **Max. Marks: 60**

I Answer any six questions $6 \times 2 = 12$

1. If $\vec{r} = 2\cos t\,\hat{i} - 3\sin t\,\hat{j} + 5\hat{k}$ *find* $\left|\frac{d^2\vec{r}}{dt^2}\right|$ *at* $t = \frac{\pi}{2}$
2. Find the unit tangent vector at $t = 1$ to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$
3. If $\phi = x^2 - y^2 + 4z$ show that $\nabla^2\phi = 0$.
4. Show that the vector $\vec{F} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$ is solenoidal.
5. Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$
6. State the Divergence theorem.
7. Find the angle between the planes 3x-6y+2z+5=0 and 4x-12y+3z-3=0.
8. Find the equation of the sphere which passes through (-1,2,3) and has its center at (3,-1,1).

II Answer any three questions $3 \times 4 = 12$

9. If $\vec{r} = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\,\hat{k}$ *find* $\left[\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \frac{d^3\vec{r}}{dt^3}\right]$
10. Derive an expression for curvature and torsion in terms of the derivative of $\vec{r}$ with arc length s.
11. For the curve $x = a\cos t, y = a\sin t, z = bt$ *ST* $\kappa = \frac{a}{a^2+b^2}$, $\tau = \frac{b}{a^2+b^2}$
12. ST the spherical coordinate system is an orthogonal curvilinear coordinate system
13. Express the vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical polar coordinates

III Answer any three questions $3 \times 4 = 12$

14. Find the directional derivative of $\emptyset = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $xlogz - y^2 = -4$ at the point (-1,2,1).
15. Show that $div\left\{r\nabla\left(\frac{1}{r^3}\right)\right\} = \frac{3}{r^4}$.

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16. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ show that $\nabla^2(r^{n+1}) = (n+1)(n+2)r^{n-1}$.

17. Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

18. Prove that div(curl($\vec{F}$)) = 0 and curl(grad(∅)) = 0.

IV Answer any three questions $3 \times 4 = 12$

19. Evaluate $\iint_R xy\, dxdy$ where R is the region bounded by the X-axis, the ordinates x=2a and the parabola $x^2 = 4ay,\ a > 0$.

20. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dxdy$ by changing the order of integration.

21. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.

22. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals.

23. Evaluate using Green's theorem $\oint_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is the square with vertices (0,0), (2,0), (2,2) *and* (0,2).

V Answer any three questions $3 \times 4 = 12$

24. Find the length and foot of the perpendicular from the point (1,1,2) to the plane 2x-2y+4z+5=0.

25. Find the equation of the planes bisecting the angle between the planes x+2y+2z=19 and 4x-3y+12z+3=0 and specify the one which bisects the acute angle.

26. Find the equations of the line passing through the point (-1,3,-2) and perpendicular to each of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ *and* $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.

27. Show that the planes 2x-2y+z+12=0 touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact

28. Find the equation of the sphere that passes through the two points (0,3,0), (-2,-1,-4) and cuts orthogonally the two spheres $x^2+y^2+z^2+x-3z-2=0$ and $2(x^2+y^2+z^2)+x+3y+4=0$.

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Vector Calculus and Analytical Geometry

Model Paper – 3

Time: 2 Hours and 30 Minutes **Max. Marks: 60**

I **Answer any six questions** $6 \times 2 = 12$

1. If $\vec{r} = \vec{a}\cos\omega t + \vec{b}\sin\omega t$ ST $\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$
2. For the space curve $\vec{r} = 3\cos t\,\hat{i} + 3\sin t\,\hat{j} + 4t\hat{k}$ find unit tangent vector at $t = \pi$
3. If $r = |\vec{r}|$ $where$ $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\nabla r^n = nr^{n-2}\vec{r}$.
4. Find div $\vec{F}$ for $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$
5. Evaluate $\int_c (x + y)dx + (y - x)dy$ along the line joining the points (1,1)and (4,2).
6. State Green's theorem in the plane.
7. Show that the planes x+2y-3z+4=0 and 4x+7y+6z+2=0 are perpendicular.
8. Find the centre and radius of the sphere $4x^2+4y^2+4z^2-16x-24y+43=0$.

II **Answer any three questions** $3 \times 4 = 12$

9. If $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, $\vec{b} = sint\hat{i} - cost\hat{j}$, $Find$ $\frac{d}{dt}(\vec{a}.\vec{b})$ and $\frac{d}{dt}(\vec{a} \times \vec{b})$.
10. Derive an expression for curvature & torsion in terms of derivatives of $\vec{r}$ w.r.t. the parameter u where $\vec{r} = \vec{r}(u)$ is the equation of the curve.
11. For the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$ $find$ $\kappa, \rho, \hat{n}, \hat{b}$.
12. Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical polar coordinates.
13. Express the vector $\vec{f} = x\hat{i} + y\hat{j} + z\hat{k}$ in spherical polar coordinates.

III **Answer any three questions** $3 \times 4 = 12$

14. Find the equations of the tangent plane and the normal line to the surface $x^3+y^3+3xyz = 3$ at the point (1,2,-1).
15. If n is a non-zero constant, show that $\nabla^2 r^n = n(n + 1)r^{n-2}$. Deduce that when $r \neq 0, r^n$ is harmonic if and only if n=-1.
16. Find curl curl $\vec{F}$ if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$,
17. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that $r^n\vec{r}$ is irrotational vector for any value of n but is solenoidal only when n=-3.

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18. Prove that $\text{div}(\vec{f} \times \vec{g}) = \vec{g}.curl\vec{f} - \vec{f}.curl\vec{g}$

IV Answer any three questions $3 \times 4 = 12$

19. If R is the region bounded by the line $x = 1, y = 0$ and the parabola $y = x^2$, evaluate $\iint_R (x^2 + y^2)dxdy$.

20. Evaluate $\iint_D \frac{x^2y^2}{x^2+y^2}\, dxdy$ where D is the annular region between the circles $x^2 + y^2 = 4$ And $x^2 + y^2 = 1$ by changing to polar coordinates.

21. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dxdydz}{(1+x+y+z)^3}$

22. Evaluate $\iiint_R xyzdxdydz$ where R is the region in the first octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing to spherical polar coordinates.

23. Evaluate by using Stoke's theorem $\oint_C (x + y)dx + (2x - z)dy + (y + z)dz$ where C is the boundary of the traingle with vertices $(2,0,0),(0,3,0)$ and $(0.0.6)$.

V Answer any three questions $3 \times 4 = 12$

24. Find the distance of the point (3, 2,1) from the plane containing the points(1,1,0). (3,- 1,1) and (-1,0,2).

25. Find k such that the lines $\frac{x-1}{2} = \frac{y-2}{2k} = \frac{z+1}{-1}$ and $\frac{x+1}{k} = \frac{y+1}{4} = \frac{z-2}{1}$ are (i) parallel and (ii) perpendicular.

26. Find the length of the perpendicular from (6,-4,4) to the line joining the points (2,1,2) and (3,-1,4). Also find its equation.

27. Find the equation of the sphere which passes through the points (1,2,3), (0,-2,4), (4,-4,2) And (3,1,4).

28. Define Orthogonal spheres and derive the condition for two spheres $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ and $x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ to be orthogonal.

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