

BENGALURU CITY UNIVERSITY
V Semester B.Sc Degree Examination
Mathematics (Core) - MATDSCT 5.2
Vector Calculus and Analytical Geometry
Model Paper - 1

Time: 2 Hours and 30 Minutes

Max. Marks: 60

I Answer any six questions

6 × 2 = 12

1. If $\vec{r} = t\hat{i} - t^2\hat{j} + \sin t\hat{k}$, find $\frac{d\vec{r}}{dt}$, $\left|\frac{d\vec{r}}{dt}\right|$ at $t = 0$
2. Define curvature and torsion of a space curve
3. Find the unit normal vector to the surface $x^2 - y^2 + z = 2$ at $(1,1,1)$
4. Show that the vector $\vec{F} = (6xy - z^2)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational.
5. Evaluate $\int_c 5xy \, dx + y^2 \, dy$ where c is the curve $y = 2x^2$ from $(0,0)$ to $(1,2)$.
6. State the Stokes theorem.
7. Find the distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$.
8. Find the equation of the sphere whose center at $(2, -3, 4)$ and radius is 5 units.

II Answer any three questions

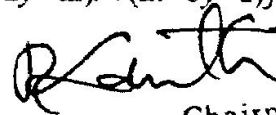
3 × 4 = 12

9. If $\vec{u} = t\hat{i} + 2t^2\hat{k}$, $\vec{v} = t^3\hat{j} + t\hat{k}$, $\vec{w} = \hat{i} + t\hat{j} + t^2\hat{k}$ Find $\frac{d}{dt}(\vec{u} \times (\vec{v} \times \vec{w}))$
10. State and prove Serenet-Frenet formula.
11. For the space curve $x = ae^u$, $y = ae^{-u}$, $z = \sqrt{2} au$ ST $\kappa = -\tau$
12. Show that the cylindrical coordinate system is orthogonal curvilinear coordinate system.
13. Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in terms of spherical polar coordinates.

III Answer any three questions

3 × 4 = 12

14. Prove that the surfaces $4x^2y + z^3 = 4$ and $5x^2 - 2yz = 9x$ intersect orthogonally at the point $(1, -1, 2)$.
15. If $f = 2xi + 3yj + 4zk$ and $\phi = xy^2z^3$ find $\vec{f} \cdot \nabla\phi$ and $|\nabla f|^2$.
16. Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$
17. Find the constants a, b, c so that the vector $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.



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18. If ϕ and \vec{F} are scalar point, vector point functions respectively then, prove that

$$\text{curl}(\phi\vec{F}) = \phi\text{curl}\vec{F} + (\text{grad}\phi) \times \vec{F}$$

IV Answer any three questions

3 × 4 = 12

19. Evaluate $\iint_R xy(x+y)dxdy$, where R is the region bounded between the parabola $y = x^2$ and the line $y = x$.
20. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by using double Integration.
21. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} xyz dx dy dz$.
22. Find the volume of the Tetrahedron bounded by the plane $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
23. Evaluate using Gauss divergence theorem $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelepiped $x = 0, y = 0, z = 0$ and $x = a, y = b, z = c$.

V Answer any three questions

3 × 4 = 12

24. Find the distance between the parallel planes $2x-y+3z+4=0$ and $6x-3y+9z-3=0$.
25. Find the image of the point $(-3,0,1)$ in the plane $4x-3y+2z=19$.
26. Find the distance between the lines $\frac{x-2}{3} = \frac{y+1}{0} = \frac{z-3}{-1}$ and $\frac{x+1}{3} = \frac{y-2}{0} = \frac{z+4}{-1}$.
27. Find the equation of the sphere which passes through the points $(1,0,0), (0,1,0), (0,0,1)$ and whose center lies on the plane $3x-y+z=2$.
28. Find the equation of the tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y-z=0$.



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Vector Calculus and Analytical Geometry
Model Paper – 2

Time: 2 Hours and 30 Minutes

Max. Marks: 60

I Answer any six questions

6 × 2 = 12

1. If $\vec{r} = 2 \cos t \hat{i} - 3 \sin t \hat{j} + 5 \hat{k}$ find $\left| \frac{d^2 \vec{r}}{dt^2} \right|$ at $t = \frac{\pi}{2}$
2. Find the unit tangent vector at $t = 1$ to the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$
3. If $\phi = x^2 - y^2 + 4z$ show that $\nabla^2 \phi = 0$.
4. Show that the vector $\vec{F} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.
5. Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$
6. State the Divergence theorem.
7. Find the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$.
8. Find the equation of the sphere which passes through $(-1, 2, 3)$ and has its center at $(3, -1, 1)$.

II Answer any three questions

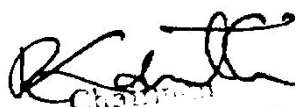
3 × 4 = 12

9. If $\vec{r} = a \cos t \hat{i} + a \sin t \hat{j} + at \tan \alpha \hat{k}$ find $\left[\frac{d\vec{r}}{dt}, \frac{d^2 \vec{r}}{dt^2}, \frac{d^3 \vec{r}}{dt^3} \right]$
10. Derive an expression for curvature and torsion in terms of the derivative of \vec{r} with arc length s .
11. For the curve $x = a \cos t, y = a \sin t, z = bt$ ST $\kappa = \frac{a}{a^2 + b^2}, \tau = \frac{b}{a^2 + b^2}$
12. ST the spherical coordinate system is an orthogonal curvilinear coordinate system
13. Express the vector $\vec{f} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical polar coordinates

III Answer any three questions

3 × 4 = 12

14. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at the point $(-1, 2, 1)$.
15. Show that $\text{div} \left\{ r \nabla \left(\frac{1}{r^3} \right) \right\} = \frac{3}{r^4}$.


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16. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ show that $\nabla^2(r^{n+1}) = (n+1)(n+2)r^{n-1}$.
17. Show that $\vec{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.
18. Prove that $\text{div}(\text{curl}(\vec{F})) = 0$ and $\text{curl}(\text{grad}(\phi)) = 0$.

IV Answer any three questions

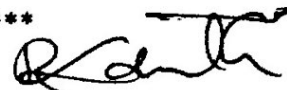
3 × 4 = 12

19. Evaluate $\iint_R xy \, dx dy$ where R is the region bounded by the X-axis, the ordinates $x=2a$ and the parabola $x^2 = 4ay$, $a > 0$.
20. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dx dy$ by changing the order of integration.
21. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dx dy dz}{\sqrt{a^2-x^2-y^2-z^2}}$.
22. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ using triple integrals.
23. Evaluate using Green's theorem $\oint_C (x^2 - xy^3)dx + (y^2 - 2xy)dy$ where C is the square with vertices (0,0), (2,0), (2,2) and (0,2).

V Answer any three questions

3 × 4 = 12

24. Find the length and foot of the perpendicular from the point (1,1,2) to the plane $2x-2y+4z+5=0$.
25. Find the equation of the planes bisecting the angle between the planes $x+2y+2z=19$ and $4x-3y+12z+3=0$ and specify the one which bisects the acute angle.
26. Find the equations of the line passing through the point (-1,3,-2) and perpendicular to each of the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.
27. Show that the planes $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact
28. Find the equation of the sphere that passes through the two points (0,3,0), (-2,-1,-4) and cuts orthogonally the two spheres $x^2+y^2+z^2+x-3z-2=0$ and $2(x^2+y^2+z^2)+x+3y+4=0$.



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Model Paper – 3

Time: 2 Hours and 30 Minutes

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6 × 2 = 12

I Answer any six questions

1. If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$ ST $\frac{d^2 \vec{r}}{dt^2} = -\omega^2 \vec{r}$
2. For the space curve $\vec{r} = 3 \cos t \hat{i} + 3 \sin t \hat{j} + 4t \hat{k}$ find unit tangent vector at $t = \pi$
3. If $r = |\vec{r}|$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $\nabla r^n = nr^{n-2}\vec{r}$.
4. Find $\text{div } \vec{F}$ for $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$
5. Evaluate $\int_C (x+y)dx + (y-x)dy$ along the line joining the points (1,1) and (4,2).
6. State Green's theorem in the plane.
7. Show that the planes $x+2y-3z+4=0$ and $4x+7y+6z+2=0$ are perpendicular.
8. Find the centre and radius of the sphere $4x^2+4y^2+4z^2-16x-24y+43=0$.

3 × 4 = 12

II Answer any three questions

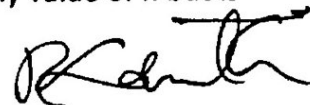
9. If $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$, $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$, Find $\frac{d}{dt}(\vec{a} \cdot \vec{b})$ and $\frac{d}{dt}(\vec{a} \times \vec{b})$.
10. Derive an expression for curvature & torsion in terms of derivatives of \vec{r} w.r.t. the parameter u where $\vec{r} = \vec{r}(u)$ is the equation of the curve.
11. For the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$ find $\kappa, \rho, \hat{n}, \hat{b}$.
12. Express the vector $\vec{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical polar coordinates.
13. Express the vector $\vec{f} = x\hat{i} + y\hat{j} + z\hat{k}$ in spherical polar coordinates.

3 × 4 = 12

III Answer any three questions

14. Find the equations of the tangent plane and the normal line to the surface $x^3+y^3+3xyz = 3$ at the point (1,2,-1).
15. If n is a non-zero constant, show that $\nabla^2 r^n = n(n+1)r^{n-2}$. Deduce that when $r \neq 0$, r^n is harmonic if and only if $n=-1$.
16. Find $\text{curl curl } \vec{F}$ if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$,
17. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that $r^n \vec{r}$ is irrotational vector for any value of n but is solenoidal only when $n=-3$.

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18. Prove that $\text{div}(\vec{f} \times \vec{g}) = \vec{g} \cdot \text{curl} \vec{f} - \vec{f} \cdot \text{curl} \vec{g}$

IV Answer any three questions

3 × 4 = 12

19. If R is the region bounded by the line $x = 1, y = 0$ and the parabola $y = x^2$, evaluate $\iint_R (x^2 + y^2) dx dy$.

20. Evaluate $\iint_D \frac{x^2 y^2}{x^2 + y^2} dx dy$ where D is the annular region between the circles $x^2 + y^2 = 4$

And $x^2 + y^2 = 1$ by changing to polar coordinates.

21. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dx dy dz}{(1+x+y+z)^3}$

22. Evaluate $\iiint_R xyz dx dy dz$ where R is the region in the first octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing to spherical polar coordinates.

23. Evaluate by using Stoke's theorem $\oint_C (x+y) dx + (2x-z) dy + (y+z) dz$ where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).

V Answer any three questions

3 × 4 = 12

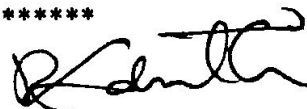
24. Find the distance of the point (3, 2, 1) from the plane containing the points (1, 1, 0), (3, -1, 1) and (-1, 0, 2).

25. Find k such that the lines $\frac{x-1}{2} = \frac{y-2}{2k} = \frac{z+1}{-1}$ and $\frac{x+1}{k} = \frac{y+1}{4} = \frac{z-2}{1}$ are (i) parallel and (ii) perpendicular.

26. Find the length of the perpendicular from (6, -4, 4) to the line joining the points (2, 1, 2) and (3, -1, 4). Also find its equation.

27. Find the equation of the sphere which passes through the points (1, 2, 3), (0, -2, 4), (4, -4, 2) and (3, 1, 4).

28. Define Orthogonal spheres and derive the condition for two spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ to be orthogonal.



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