K.L.E Society's
S. Nijalingappa College
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PG Department of Mathematics
QUESTION BANK

Numerical Analysis:

1. Make a discussion on the absolute stability of the classical Runge-Kutta explicit mehod of one slope.
2. Find four iterations of the Picard method the solution y(0, 1) of the problem: $dy/dx=x+ y^2$; y(0)=1.
3. Show that the Runge-Kutta methods are relatively stable.
4. Solve dy/ dx $=x+y^2$: y (0) =1 by any Runge-Kutta method of two slopes. Choose $\Delta x =$ 0.05 and obtain y (0.1).
5. Adam's predictor-corrector method of any order. Using the method solve $dy/dx =x+y^2$; y(0)=1,Obtain y(0.6). Generate needed initial values by any single-step method.
6. Solve the boundary value problem $x\frac{d^2y}{dx^2} + y = x^2, y(0) = 1, y(1) = 1$ by both shooting and cubic-spline methods. Choose an appropriate ofΔx.
7. Establish Picard iteration method for $y' = f(x, y), y(x_0) = y_0$.find the first four successive iteration of$y' = x + y$, y (0)=1 and obtain at y(0.5).
8. Solve $\frac{dy}{dx} = xy;\ \ y(1) = 2$ find y (1.4) with h = 0.2 using Runge- Kutta method of fourth order.
9. Use Picard's method to obtain $\frac{dy}{dx} = 1 + xy;\ \ y(0) = 1$ Find y(0.1).
10. Solve $(x + 1)y'' + y = 0;\ y(0) = 1, y(0.2) = 0$ by the shooting method.
11. Solve the BVP $xy'' + x^2y' + 6y = sinx$, $0 \le x \le 2;\ y(0) = 1,\ y(2) = 1$ with $h =$ 0.5 using finite difference method.
12. Solve the differential equation by Galerkin method $y'' + y = -1;\ y(0) = 1, y(1) = 1$ with$x = 0.5$.
13. Derive Runge-Kutta method of second order.

14. Solve $\frac{dy}{dx} = z - y$; $y(0) = 1$ and $\frac{dz}{dx} = y - z$; $z(0) = 0$ by classical Runge-Kutta method of second order. Choose $\Delta x = 0.05$. Obtain the solution at $x = 0.05$

15. Using finite difference method, solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subjected to the conditions

$$\left.\begin{matrix} u(x,0) = x \\ u(x,1) = 0 \end{matrix}\right\} \quad 0 \le x \le 1 \quad \left.\begin{matrix} u(0,y) = 0 \\ u(1,y) = 1 \end{matrix}\right\} \quad 0 \le y \le 1 \text{ take } \Delta x = \Delta y = 1/3.$$

16. Using finite difference method, solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \sin\pi x \sin\pi y$ subjected to the conditions.U=0 on the boundaries take$\Delta x = \Delta y = 1/3$

17. Using Schmidt explicit formula, solve one - dimensional heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subjected to the conditions,

$$u(x,0) = x(1-x) \quad 0 \le x \le 1 \quad \left.\begin{matrix} u(0,y) = 0 \\ u(1,y) = 1 \end{matrix}\right\} \quad 0 \le y \le 1$$

Where h=$\frac{1}{4}$& k =$\frac{1}{32}$, obtain the solution at second – time level.

18. Derive the stability condition for Crank – Nicolson method to solve one - dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

19. Solve the equation$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial t^2}$ subjected to the boundary conditions $u(x,0) = x(1-x)$, $u(0,t) = u(1,t) = 0$ Using Crank- Nicolson method for first time level.

20. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2}$ subjected to the boundary conditions $u(x,0) = x(1-x)$, $u_t(x,0) = 0, u(0,t) = u(4,t) = 0$, $0 \le x \le 4, 0 \le t \le \infty$ with h=1, k = 0.5 obtain the solution at second time level using explicit formula

21. Obtain the stability of wave equation (implicit form).

22. Derivethe alternating direction method and hence show that the method is unconditionally stable.

23. Find the solution of $U_{tt} = U_{xx} + U_{yy}; 0 \le x, y \le 1$ with the boundary condition U(x, y,0)= sinπxsin$\pi y U_t(x,y,0) = 0$ and U=0 on the boundaries take $\Delta x = \Delta y = 1/3$, $\Delta t = 1/9$ at second time level.