K.L.E Society's
S. Nijalingappa College
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PG Department of Mathematics
QUESTION BANK

Partial Differential Equations

1. Define the Cauchy problem for first order partial differential equation with its geometrical representation. Further explain the method of characteristics for solving semi linear PDE.
2. Solve: $uu_x + u_y = 0$ with $u = x$ when $y = 0$.
3. Solve: $u_t + u\,u_x = 0$ with $u(x,0) = \begin{cases} 1 & \text{if } x \leq 0 \\ 1-x & \text{if } 0 \leq x \leq 1 \\ 0 & \text{if } x \geq 1. \end{cases}$
4. Find the complete integral of $z = pq$.
5. Solve: $pq - u = 0$ with $u = y^2$ when $x = 0$.
6. Classify the second order partial differential equation into hyperbolic, parabolic and elliptic and hence reduce it to canonical form for hyperbolic.
7. Transform $x^2 u_{xx} - 2xy\, u_{xy} + y^2\, u_{yy} = e^x$ into its canonical form.
8. Solve:

   (i) $x^2 r - y^2 t = x^2 y^3$.

   (ii) $(D^2 - D'^2 + 3D' - 3D)\, u = 0$.

9. Solve: $xy(t - r) + (x^2 - y^2)(s - 2) = py - qx$ by using Monge's method.
10. Solve the following by using the method of separation of variables:

$$u_{tt} = c^2 u_{xx};\ 0 \leq x \leq 1, t \geq 0 \text{ subject to}$$

$$\left.\begin{aligned} u(x,0) &= f(x) \\ u_t(x,0) &= g(x) \end{aligned}\right\};\ 0 \leq x \leq 1$$

$$u(0,t) = 0 = u(1,t);\ t \geq 0$$

11. Using an appropriate Fourier transform solve the following:

$$u_{tt} = c^2 u_{xx};\ -\infty < x < \infty, t \geq 0 \text{ subject to}$$

$$\left.\begin{aligned} u(x,0) &= f(x) \\ u_t(x,0) &= 0 \end{aligned}\right\};\ -\infty < x < \infty$$

12. State and solve Neumann problem for a rectangle.
13. State and solve exterior Dirichlet problem for a circle.
14. Using Duhamel's principle solve the following:

$u_t = u_{xx}; 0 < x < 1, t \geq 0$

subject to

$u(x,0) = 0;\ \ 0 < x < 1$

$$\left.\begin{aligned} u(0,t) &= 0 \\ u(1,t) &= f(t) \end{aligned}\right\} ;\ t \geq 0$$

15. Find the solution of three-dimensional heat equation in cylindrical polar coordinates.

16. Determine the Green's function for $u_t = \alpha\, u_{xx}; -\infty < x < \infty, t \geq 0$
subject to $u(x,0) = f(x)\,; -\infty < x < \infty$

17. Find the Green's function for the following:

$$-\left[u_{xx} - \frac{1}{c^2} u_{tt}\right] = q(x,t); -\infty < x < \infty; t > 0$$

subject to

$$\left.\begin{aligned} u(x,0) &= 0 \\ u_t(x,0) &= 0 \end{aligned}\right\} ; -\infty < x < \infty$$

$$\left.\begin{aligned} u &\to 0 \\ \frac{\partial u}{\partial x} &\to 0 \end{aligned}\right\} ; \text{as } |x| \to \infty$$

18. (a) Explain the method of characteristic for solving quasi linear partial differential equation.

(b)Solve: $u_x + u_y + u = 1$ with $u = \sin x$ on $y = x + x^2$.

(c)Find the complete integral of $px + qy = pq$.

19. (a) Solve: $u_t + u\,u_x = 0$ with $u(x,0) = \begin{cases} 1 & \text{if } x \leq 0 \\ 1 - 2x & \text{if } 0 \leq x \leq 1 \\ -1 & \text{if } x \geq 1 \end{cases}$

20. Solve: $p^2 - 3q^2 - u = 0$ with $u(x,0) = x^2$.

21. Classify the second order partial differential equation into hyperbolic, parabolic and elliptic and hence reduce it to canonical form for parabolic.

22. Transform $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x\, u_{yy} - \cos x\, u_y = 0$ into its canonical form.

23. Solve: (i) $\left(D^3 - 7DD'^2 - 6D'^3\right) u = \sin(x + 2y) + e^{x+2y}$.

(ii) $(D^2 - 2DD' + D'^2)\, u = e^{x+2y}$.

24. Explain the Monge's method for solving the equation $Rr + Ss + Tt = V$.

25. Solve:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \pi^2 \sin(\pi x); 0 \le x \le 1, t \ge 0$$

subject to

$$u(0,t) = u(1,t) = 0; t \ge 0$$

$$\left.\begin{aligned} u(x,0) &= \sin(3\pi x) \\ \frac{\partial u(x,0)}{\partial t} &= 0 \end{aligned}\right\}; 0 \le x \le 1$$

26. Using an appropriate Fourier transform solve the following:

$u_{tt} = c^2 u_{xx}; 0 \le x \le L, t \ge 0$ subject to

$$\left.\begin{aligned} u(x,0) &= f(x) \\ u_t(x,0) &= g(x) \end{aligned}\right\}; 0 \le x \le L \text{ and}$$

$$u_x(0,t) = 0 = u_x(L,t); t \ge 0.$$

27. Find the solution of the three – dimensional Laplace equation in cylindrical polar coordinates.

28. State and solve Dirichlet problem for a rectangle..

29. Solve the following by using the method of separation of variables:

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial x^2}; 0 \le x \le a, t \ge 0$$

subject to

$$u(x,0) = f(x); 0 \le x \le a$$

$$u(0,t) = 0 = u(a,t); t \ge 0$$

30. Solve the following by using an appropriate Fourier transform.

$$\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}; -\infty < x < \infty, t \ge 0$$

subject to $u(x,0) = f(x); -\infty < x < \infty$

31. Find the Green's function for the Dirichlet problem on the rectangle $\mathbb{R}$:

$0 \le x \le a, 0 \le y \le b$, described by the PDE $(\nabla^2 + \lambda)\mathrm{u} = 0$ in $\mathbb{R}$.

32. Find the Green's function for the following:

$$u_t = \alpha u_{xx}; 0 \le x \le L, t > 0$$

subject to $u(0,\ t) = u(L,t) = 0; t > 0$

$$u(x,0) = f(x);\ 0 \le x \le L$$