K.L.E Society's
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QUESTION BANK

Riemannian Geometry:

1. a) Define Smooth manifold. The subset $M \subset R^2$ defined by M $=\{(\sin 2\pi s, \sin \pi s), S \in M\}$ is called figure-8 and M is not adifferential Manifold

   b) Define Tangent space. $\Phi : M \to N$ be a c^{∞} map and f,g ∈ C then show that

   i. $\Phi^*(f+g) = \Phi^* f + \Phi^* g$
   ii. $\Phi^*(fg) = \Phi^* f . \Phi^* g$
   iii. $\Phi^*(\gamma f) = \gamma \Phi^* f \quad \gamma \in R$

   OR

   b) Define smooth maps. $\Phi: M \to M'$ and $\varphi : M' \to M''$ are C map then show that $(\varphi\ o\ \Phi)^* = \Phi^*\, o\ \varphi^*$

2. a) If $X = yz\frac{\partial}{\partial x}$, $Y = zx\frac{\partial}{\partial y}$, $Z = xy\frac{\partial}{\partial z}$ then verify the Jacobi Identity for Lie bracket of vector fileds.

   b) Define Riemannian connection. Ad prove that R is a field of type (1,3)

3. Define Diffiomorphism. And show that $[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0$ as a Jocobi identity

4. Explain Jacobi Map. Let $\Phi: R^2 \to R^3$ $\Phi(x,y) = (u^2 v + v^2, u - 2v^3, -ev^u)$. Evaluate $\Phi_{*(0,1)} = \left(4\frac{\partial}{\partial u} - \frac{\partial}{\partial v}\right)$

5. State and Prove Bainchi 2^{nd} Identity

6. For the components Γ_{ij}^h of a remannian connection ∇ on ,anifold of (M,g)

   i. $\partial_k g_{ij} = g_{hj}\Gamma_{ki}^h + g_{hi}\Gamma_{kj}^h$
   ii. $\Gamma_{ki}^h = \partial_j log\sqrt{g}$ where $g = |g_{ij}|$

7. State and prove Fundamental theorem of Remannian Geometry

   OR

   Define Cotangent Tensors. Let $T, S \in D_s^r . then$ $\nabla_X(T+S) = \nabla_X T + \nabla_X S$,
   Let $T \in D_s^r(M), S \in D_q^p(M), f \in c^{\infty}$ $\alpha \in R$. Then

   i. $\nabla_X(T \otimes S) = (\nabla_X T) \otimes S + S \otimes (\nabla_X S)$
   ii. $\nabla_X(fT) = (Xf)T + f(\nabla_X T)$

8. State and Prove Bainchi First Identity

   OR

   Define tensors on Manifolds and explain the followings

   i. Component of tensors
   ii. Transformation formula for components of Tensors.