K.L.E Society's
S. Nijalingappa College
II BLOCK RAJAJINAGAR, BENGALURU -10

<u>PG Department of Mathematics</u>
QUESTION BANK

TOPOLOGY-I

1. State and prove Schroder- Bernstein Theorem.
2. (a) Define topological Space. In topological space (X, τ) show that
   i. Arbitrary intersection of closed sets is closed.
   ii. Finite Union of closed sets is closed.

   (b) Define limit point of a topological space. If X ={ a, b, c } ,τ ={ Φ, X, {a},{b, c} } i. If A={c} find d (A).

   ii. Find d ({a, b}), d({b ,c}), d({a, c}).
3. (a) Let (X,τ) be any topological space, then for any set A,B $\subseteq$ X. show that
   i. d(Φ)=Φ
   ii. A$\subseteq$B $\Rightarrow$ d(A) $\subseteq$ d(B)
   iii. d(A$\cup$B) = d(A)$\cup$d(B)
4. (a) Define closure of a set. In a topological space (X, τ) for any A$\subseteq$X. show that A$\cup$d(A) is closed.

   (b) Define Interior of a set. For any A$\subseteq$X, show that $A^{o} = (\bar{A}^{1})^{1}$
5. (b) Define the continuity of a function in topological space. $f :\to Y$ is continuous at x iff V is a neighbourhood of f(x) $\Rightarrow$ f^{-1}(V) is nbd of x.

   (b) State and prove pasting lemma.
6. (a) Define Homeomorphism. A bijective function $f{:}X \to Y$ is a Homeomorphism, iff f($\bar{A}$) $\subseteq$ $\overline{f(A)}$ for any set A$\subseteq$X.

7. Define bases of a topological space with an example. And Prove that is subfamily β of τ is a base for τ iff U$\in\tau$ and x$\in$U implies there exists B$\in\beta$ such that x$\in$B$\subseteq$U.

8. Define Cauchy's Sequence. Let X be a complete space and Y be a subspace of X the Y is complete iff it is closed
9. State and prove cantor's intersection theorem.
10. State and prove contraction mapping theorem.

11. State and prove Bair’s category theorem.

12. Define limit point of a topological space. If $X = \{ a, b, c \}$, $\tau = \{ \Phi, X, \{a\}, \{b, c\} \}$ If $A = \{c\}$ find d (A).

13. Let $A \subseteq (X,d)$ then followings are equivalent

i) X-A is Open

ii) $d(A) \subseteq A$ Where d(A) is the set of all limit points of A **(2+3)**

14. Define topological Space. In topological space (X, τ) show that

i. Arbitrary intersection of closed sets is closed.

ii. Finite Union of closed sets is closed.