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BENGALURU CITY UNIVERSITY

V SEMESTER B.Sc, MATHEMATICS, PAPER – 5 (CBCS 2020 onwards)

Model Paper 1

Time: 3 hrs — Max marks: 70

I. Answer any FIVE questions $2 \times 5 = 10$

1). In a ring (R,+,.) prove that $a.0 = 0 = 0.a$

2). Define a subring with an example

3). Define Quotient ring.

4). What is a Geodesic on the plane.

5). Write the Euler's formula to find the extremum of a functional $\int_{x_1}^{x_2} f(x, y, y')dx$

6).Prove that $\nabla\Delta = \Delta - \nabla$

7).Write Newton backward interpolation formula

8). Write the Trapezoidal rule for numerical Integration.

II. Answer any TWO questions $2 \times 5 = 10$

9). Prove that $(Z_5, +_5, \times_5)$ is a commutative ring with respect to addition modulo 5 and multiplication modulo 5.

10). Prove that every finite Integral Domain is a field.

11). Prove that intersection of two Ideals is an Ideal. Is the union an Ideal. Give an example.

III. Answer any TWO questions $2 \times 5 = 10$

12). Find the extremal of the functional $I = \int_0^{\frac{\pi}{2}} (y^2 - y'^2 - 2y\sin x)dx, y(0) = 0, y\left(\frac{\pi}{2}\right) = 0.$

13). Find the path in which a particle in the absence of friction moves from one point to another in shortest time

14). Show that the extremal of the functional $\int_0^1 y'^2 dx$ subject to the constraint $\int_0^1 y dx = 1$ having $y(0) = 0, y(1) = 1$ is a parabolic curve.

IV. Answer any THREE questions $3 \times 5 = 15$

15) Prove that $(i) E\nabla = \nabla E = \Delta$ $(ii) \left(\frac{\Delta^2}{E}\right) x^3 = 6xh$

16). The values of $\sin x$ are given below for different values of x. Find the value of $\sin 32^0$

x	30^0	35^0	40^0	45^0
y=sinx	0.5	0.5736	0.6428	0.7071

[signature]

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

17). Find $f(2)$ using Lagrange interpolation formula

x	0	1	3	4
f(x)	5	6	50	105

18). Find the equation of the cubic curve which passes through the points (4, -43), (7, 83), (9,327) (12, 1053) Find f (10) by using divided difference formula

19). Evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ by using Simpson's $\frac{3}{8}^{th}$ rule

V. Answer any THREE questions $3 \times 5 = 15$

20). If $f: R \to R'$ be a homomorphism from R onto R'. Then show that ker(f) is an ideal of R.

21). Show that the extremal of the functional $I = \int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is expressible in the form of $y = ae^{bx}$

22). Find the curve which makes the functional $\int_0^1 (y'^2 + x)dx$ an extremum. Given $y(0) = 0, y(1) = 1$ Under the constraint $\int_0^1 y dx = 1$

23).Obtain the function whose first difference is $6x^2 + 10x + 11$

24) Find $\int_0^1 \frac{x^2}{1+x^3} dx$ by using Simpson's $\frac{1}{3}^{rd}$ rule

VI. Answer any TWO questions $2 \times 5 = 10$

25) Light travels in a medium from (0,0) to (2a,0) such that the velocity is given by v=y. If the time taken is an extremum, find the path.

26) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines x=0 & x=1 and a curve through the points with the following coordinates

x	0	0.25	0.5	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using trapezoidal rule

27) A reservoir discharging water through sluices at a depth x below the water surface has a water surface area A for various values of x as given in the following table

x(ft)	10	11	12	13	14
A(sq. ft)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dx}{dt} = -48\left(\frac{x}{A}\right)$ Estimate the time taken for the water level to fall from 14 to 10 ft above the sluices using Simpson's $\frac{3}{8}^{th}$ rule

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Model Paper 2

Time: 3 hrs | Max marks: 70

I. Answer any FIVE questions $2 \times 5 = 10$

1). Define Commutative Ring and Ring with zero devisors with an example.

2) Define left ideal and right ideal of a Ring (R,+,.)

3).Define Homomorphism of rings

4). State Isoperimetric problem in Calculus of variations.

5). If u, v are functions of x, y, y' prove that $\delta(uv) = u\delta v + v\delta u$

6) Prove that $\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)(1+\Delta)^{\frac{1}{2}} = 2 + \Delta$.

7) Write Newton Gregory forward interpolation formula.

8) Write Lagrange's inverse interpolation formula for unequal intervals

II. Answer any TWO questions $2 \times 5 = 10$

9). Prove that every field is an Integral Domain.

10). Show that the set of all real numbers of the form $a + b\sqrt{3}$ where a,b are integers is an Integral domain with respect to usual addition and and multiplication.

11). If R is a commutative Ring and $a \in R$,then $Ra = \{ra, r \in R\}$ is an ideal of R

III. Answer any TWO questions $2 \times 5 = 10$

12). Show that the extremal of the functional $\int_{x_1}^{x_2} \sqrt{y(1 + y'^2)}dx$ is a parabola

13). Prove that the shortest distance between two points in a plane is along a straight line joining them.

14) Find the extremal of the functional $\int_0^1 (y'^2 + x^2)dx$ subject to $\int_0^1 y dx = 2$ and having conditions $y(0) = 0, y(1) = 1$.

IV. Answer any THREE questions $3 \times 5 = 15$

15). Use the method of separation of symbols to prove

$$u_0 - u_1 + u_2 + \cdots = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \cdots$$

16). Find polynomial of degree 2 of the following data

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

17). From the data given below, find the number of students whose weights are between 80 and 90

Weight in kgs	0-40	40-60	60-80	80-100	100-120
No. of students	250	120	100	70	50

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus

18).By using Lagrange's interpolation for unequal intervals find f (6)

x	7	8	9	10
f(x)	3	1	1	63

19). Evaluate $\int_0^{\pi} \frac{dx}{2+\cos x}$ using Simpson's $\frac{3}{8}^{th}$ rule.

V. Answer any THREE questions $3 \times 5 = 15$

20). If $f: R \to R'$ is an isomorphism and if R is a Ring with unity, then prove R' is also a ring with unity.

21).Solve the variational problem $\delta \int_0^{\frac{\pi}{2}} (y^2 - y'^2)dx = 0, y(0) = 0, y\left(\frac{\pi}{2}\right) = 2$

22). Show that the surface of revolution of a given area enclosing maximum volume is a sphere

23) Express $3x^3 - 4x^2 + 3x + 1$ is factorial notation.

24) Find $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule

VI. Answer any TWO questions $2 \times 5 = 10$

25) If the cost/unit distance to travel in the first quadrant ($x \geq 0, y \geq 0$) is proportional to 1+x, find the equation of the family of curves along which travel is cheapest

26) The velocity V(m/sec) of a particle at a distance S(m) from a point on its linear path is given by the following table

S(m)	0	2.5	5	7,5	10	12.5	15
V(m/sec)	16	19	21	22	20	17	13

Estimate the time taken by the particle to traverse the distance of 15 m using Simpson's $\frac{3}{8}^{th}$ rule

27) A reservoir discharging water through sluices at a depth x below the water surface has a water surface area A for various values of x as given in the following table

x(ft)	10	11	12	13	14
A(sq. ft)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dx}{dt} = -48\left(\frac{x}{A}\right)$ Estimate the time taken for the water level to fall from 14 to 10 ft above the sluices using Simpson's $\frac{1}{3}^{rd}$ rule

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Model Paper 3

Time: 3 hrs | Max marks: 70

$2 \times 5 = 10$

I. Answer any FIVE questions

1). In a Ring(R,+,.) Prove that a.(b-c)=a.b-a.c

2). Define the principal Ideal Ring.

3). Define Isomorphism of Rings.

4). If u, v are functions of x, y, y' and c is constant, prove that $\delta(cu) = c\delta u$

5). Write the Euler's formula to find the extremum of a functional $\int_{x_1}^{x_2} f(y, y')dx$

6).Prove that $\delta = E^{\frac{1}{2}}\nabla$

7). Write Lagrange's formula for unequal interval.

8). Write the Trapezoidal rule for numerical Integration

$2 \times 5 = 10$

II. Answer any TWO questions

9). Prove that the necessary and sufficient condition for a non-empty subset S of a Ring R to be a subring are (i) $a \in S, b \in S \Rightarrow a - b \in S$ and (ii) $ab \in S$.

10). Find all Principal ideals of the Ring $R = \{0, 1, 2, 3, 4, 5\}$ with respect to $+_6, \times_6$

11). Prove that a commutative ring with unity is a field if it has no proper ideals.

$2 \times 5 = 10$

III. Answer any TWO questions

12) State and prove Euler's formula to find the extremum of a functional $\int_{x_1}^{x_2} f(x, y, y')dx$.

13) Find the extremal of the functional $I = \int_{x_1}^{x_2}(y'^2 - y^2 + 2y\sec x)dx$

14) Prove that the catenary is the curve which when rotated about a line generates a surface of minimum area.

$3 \times 5 = 15$

IV. Answer any THREE questions

15). Prove that $(i)(1 + \Delta)(1 - \nabla) = 1$ $(ii)\frac{\Delta}{\nabla} - \frac{\nabla}{\Delta} = \Delta + \nabla$

16). If $y_0 = 1, y_1 = 11, y_2 = 21, y_3 = 28, y_4 = 29$ find $\Delta^4 y_0$

17). From following data, find y when x=84

x	40	50	60	70	80	90
y	184	204	226	250	276	304

18). Find f(2) using Lagrange interpolation formula

x	5	6	9	11
f(x)	12	13	14	16

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

19). Evaluate $\int_1^4 e^{\frac{1}{x}}\,dx$ by using Simpson's $\frac{3}{8}^{th}$ rule

V. Answer any THREE questions $3 \times 5 = 15$

20). State and prove Fundamental theorem of homomorphism on Rings

21). Find the curve through (0,1) and (1,2) along which $I = \int_0^1 (y^2 - yy' + y'^2)dx$ is minimum.

22). Find the extremal of the functional $\int_0^1 (y'^2 + x^2)dx$ subject to $\int_0^1 ydx = 2$ and having conditions $y(0) = 0, y(1) = 1$.

23) Express $3x^3 + x^2 + x + 1$ is factorial notation.

24) Find $\int_1^5 \log_{10} x\, dx$ taking 8 sub intervals correct to four decimal places by using trapezoidal rule.

VI. Answer any TWO questions $2 \times 5 = 10$

25) Find the plane curve of length 'l' having end points at (x_1, y_1) & (x_2, y_2) such that the area under the curve (between $x = x_1$ & $x = x_2$) is maximum

26) The velocity v(km/min) of a moped which starts from rest is given at fixed intervals of time t(min) as follows

t	0	2	4	6	8	10	12	14	16	18	20
v	0	10	18	25	29	32	20	11	5	2	0

Estimate approximately the distance covered in 20 mins. Using Simpson's $\frac{1}{3}^{rd}$ rule

27) A reservoir discharging water through sluices at a depth x below the water surface has a water surface area A for various values of x as given in the following table

x(ft)	10	11	12	13	14
A(sq. ft)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dx}{dt} = -48\left(\frac{x}{A}\right)$ Estimate the time taken for the water level to fall from 14 to 10 ft above the sluices using trapezoidal rule

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