BENGALURU CITY UNIVERSITY

V Semester B.Sc. MATHEMATICS- PAPER 6 (CBCS 2020 onwards)

Model Paper 1

Time: 3 hrs. **Max Marks: 70**

I Answer any five questions: **(5x2=10)**

1. Find $\nabla^2\phi$ *for* $\phi = x^3y^2z$ *at* $(1,2,3)$
2. If $\vec{F} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ then show that $\vec{F}$ is irrotational
3. Evaluate $\int_c 5x\,dx + y\,dy$ where c is the curve $y = 2x^2$ *from* $(0,0)$ *to* $(1,2)$
4. Evaluate $\int_0^a \int_0^b \left(x^2 + y^2\right) dy\,dx$
5. Evaluate $\int_0^1 \int_0^2 \int_0^3 (x+y+z)\,dx\,dy\,dz$
6. Evaluate $\int_0^{\pi/2} \int_0^{a\cos\theta} r^2\,dr\,d\theta$
7. State Green's theorem
8. Evaluate $\int_c yz\,dx + zx\,dy + xy\,dz$ where c is the curve $x^2 + y^2 = 1, z = y^2$ using Stoke's theorem

II Answer any two questions: **(2x5=10)**

9. Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ *where* $r^2 = x^2 + y^2 + z^2$
10. Show that the surfaces $4x^2y + z^3 = 4$ & $5x^2y - 2yz = 9x$ intersect orthogonally at the point $(1,-1,2)$
11. If $\vec{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$ *find* $div\vec{F}$ *and* $curl\vec{F}$

III Answer any three questions: **(3x5=15)**

12. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant bounded by the circle $x^2 + y^2 = 1$
13. Show that the line integral $\int_c (x^3 + 2yz)dy + 3x^2y\,dx + y^2dz$ is independent of the path joining the points $(1,-2,3)$ & $(3,2,-1)$
14. Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{(x^2+y^2)}\,dx\,dy$

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15. Find area enclosed between the circles $r = \sin\theta \ and \ r = 4\sin\theta$

16. If R is the region bounded by the planes $x = 0, y = 0, z = 0 \ and \ x + y + z = 1$

    Evaluate $\iiint_R z\,dx\,dy\,dz$

IV Answer any two questions: (2x5=10)

17. Using Green's theorem find the area of the circle $x^2 + y^2 = a^2$

18. Using Gauss divergence theorem evaluate

    $\iint_S \vec{F}.\hat{n}\,ds \ where \ \vec{F} = 2xy\,\hat{i} + yz^2\,\hat{j} + xz\,\hat{k} \ and \ S$

    is the total surface of the cuboid bounded by the planes

    $x = 0, y = 0, z = 0, x = 1, y = 2, z = 3$

19. Evaluate by Stoke's theorem $\oint_c \sin z\,dx - \cos x\,dy + \sin y\,dz$ where c is the boundary of the Rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3$ and its boundary

V Answer any three questions: (3x5=15)

20. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \ and \ r = |\vec{r}|$ then prove that

    $a)\nabla r^n = n\,r^{n-2}\,\vec{r}$ $\qquad b)\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

21. If ϕ is a scalar function then prove that $curl\left(grad\phi\right) = 0$

22. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming to spherical polar coordinates

23. Find the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the $x -$axis

24. Using divergence theorem prove that

    $a)\iint_S \vec{r}.\hat{n}\,ds = 3V$ $\qquad b)\iint_S \nabla r^2.\hat{n}\,ds = 6V$

VI Answer any two questions: (2x5=10)

25. Find the work done by the force $\vec{F} = 4xy\,\hat{i} - 8y\,\hat{j} + 2\,\hat{k}$ in the displacement along the straight line $y = 2x, z = 2x$ from $(0,0,0) \ to \ (3,6,6)$

26. Find moment of inertia of a circular plate of mass 'm' radius 'a' about the y-axis through its centre

27. Determine the x-coordinate of centre of gravity of the plane lamina of uniform surface density bounded by the upper half of the cardioid $r = a\left(1 + \cos\theta\right)$

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Model Paper 2

Time: 3 hrs. **Max Marks: 70**

I Answer any five questions: **(5x2=10)**

1. If $\phi = x^2 - y^2 + 4z$ then prove that $\nabla^2\phi = 0$
2. Find the unit normal vector to the surface $(x-1)^2 + y^2 + (z+2)^2 = 9$ at $(3,1,-4)$
3. Evaluate $\int_c xdx + ydy$ where c is the curve $y^2 = x$ from $(1,1)$ to $(2,4)$
4. Evaluate $\int_0^1 \int_0^{1-x} dy\,dx$
5. Evaluate $\int_0^3 \int_0^2 \int_0^1 x\,y\,z\,dx\,dy\,dz$
6. Evaluate $\int_0^{\pi/2} \int_0^a r^2 dr\,d\theta$
7. State Stoke's theorem
8. Write vector form of Green's theorem

II Answer any two questions: **(2x5=10)**

9. Find the directional derivative of $\phi(x,y,z) = x^2 - y^2 + 4z^2$ at $(1,1,-8)$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$
10. If $\vec{F} = (ax+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}$ is solenoidal, find a
11. Show that $\vec{F} = (6xy+z^3)\hat{i} + (3x^2-z)\hat{j} + (3xz^2-y)\hat{k}$ is irrotational . Find ϕ such that $\vec{F} = \nabla\phi$

III Answer any three questions: **(3x5=15)**

12. Evaluate $\int_c (x+y+z)\,ds$ where c is the line joining the points $(0,1,0)$ & $(1,2,3)$
13. Evaluate $\iint_R (4x^2 - y^2)\,dx\,dy$ where R is the area bounded by the lines $y=0,\ y=x,\ x=1$
14. Find the volume underneath the surface $x+y+z=2$ which cuts the cylinder $x^2+y^2=1$ above the xy plane
15. Evaluate $\int_0^1 \int_{y^2}^{\sqrt{y}} x\,y\,dx\,dy$ by changing the order of integration

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16. Find the volume enclosed by the cylinder $x^2+y^2=a^2$ enclosed between the planes $z=0$ & $z=c$ by transforming to cylindrical polar coordinates

IV Answer any two questions: **(2x5=10)**

17. Using Green's theorem evaluate $\int_c xy\,dx + yx^2\,dy$ where c is the curve enclosing the region bounded by the curve $y=x^2$ & $y=x$
18. Using Gauss divergence theorem evaluate
$\iint_S \vec{F}.\hat{n}\,ds$ *where* $\vec{F}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ *and* S
is the surface enclosing the region for which $x^2+y^2\le 4$ & $0\le z\le 3$
19. Evaluate $\iint_S curl\vec{F}.\hat{n}\,ds$ *where* $\vec{F}=(y-z+2)\hat{i}+(yz+4)\hat{j}-xz\hat{k}$ *and* S is the surface of the cube $0\le x\le 2$, $0\le y\le 2$, $0\le z\le 2$ using Stoke's theorem

V Answer any three questions: **(3x5=15)**

20. Prove that $(a)\,curl\vec{F}$ is solenoidal $(b)\,grad\phi$ is irrotational
21. If ϕ is a scalar function and $\vec{F}$ is a vector function then prove that
$div(\phi\vec{F})=\phi\, div\vec{F}+\nabla\phi.\vec{F}$
22. Evaluate $\iint_R \frac{x^2y^2}{x^2-y^2}\,dx\,dy$ by transforming to polar coordinates where R is the region between the circles $x^2+y^2=4$ & $x^2+y^2=1$
23. Find the volume of the cylinder $x^2+y^2=a^2$ bounded by the planes
$x=0, y=0, z=0, z=b$
by transforming to cylindrical coordinates
24. State and prove Green's theorem

VI Answer any two questions: **(2x5=10)**

25. Find the work done by the force $\vec{F}=4xy\hat{i}-8y\hat{j}+2\hat{k}$ in the displacement around the circle
$x^2+y^2=4, z=0$ in the counterclockwise direction
26. Find moment of inertia of a circular plate of mass 'm' radius 'a' about the x-axis through its centre
27. Find the y-coordinate of the centre of gravity of a mass of density $f(x,y)=1$ in aregion R, where R is a rectangular plate of dimension $2x4$ *ie* $0\le x\le 2, 0\le y\le 4$

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Model Paper 3

Time: 3 hrs. **Max Marks: 70**

I Answer any five questions: **(5x2=10)**

1. If $\vec{F} = 3x^2\,\hat{i} + 5xy^2\,\hat{j} + xyz^3\,\hat{k}$ find $div\vec{F}$ at $(1,2,3)$
2. If $\vec{F} = x^2y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$ find $curl\vec{F}$
3. Evaluate $\int_0^1 \int_0^x xy\,dy\,dx$
4. Evaluate $\int_c x\,dy + y\,dx$ along the curve $y = x^2$ from $(0,0)$ to $(1,1)$
5. Evaluate $\int_{-3}^{3} \int_{-2}^{2} \int_{-1}^{1} x^2 y^2 z^2\,dz\,dy\,dx$
6. Evaluate $\int_0^{\pi/2} \int_0^{a} r\sin\theta\,dr\,d\theta$
7. State Gauss divergence theorem
8. Using Stoke's 's theorem evaluate $\oint_c \vec{r}.d\vec{r}$

II Answer any two questions: **(2x5=10)**

9. Find constants a,b,c if the vector $\vec{F} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ is irrotational
10. If n is a non zero constant then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ Also prove that r^n is harmonic if $n = -1$
11. If $\vec{F}$ and $\vec{G}$ are vector functions then prove that $\nabla.\left(\vec{F}x\vec{G}\right) = \vec{G}.curl\vec{F} - \vec{F}.curl\vec{G}$

III Answer any three questions: **(3x5=15)**

12. Evaluate $\int_c xy\,dx + yz\,dy + zx\,dz$ where c is the line joining the points $(0,0,0)\&(2,1,3)$
13. Evaluate $\iint_R (x^2+y^2)\,dx\,dy$ where R is the area bounded by the lines $x = 0,\ x = y,\ y = 1$
14. Find the area of the cardioid $r = a(1+\cos\theta)$ using double integration
15. Find volume of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming to cylindrical polar coordinates

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16. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\,dy\,dx}{\sqrt{1-x^2-y^2-z^2}}$

IV Answer any two questions: (2x5=10)

17. Using Green's theorem evaluate $\int_c y^2\,dx + x^2\,dy$ where c is the closed curve bounded by $y = x$ & $y^2 = x$

18. Using Gauss divergence theorem evaluate $\iint_S \vec{F}.\hat{n}\,ds$ *where* $\vec{F} = x^3\,\hat{i} + y^3\,\hat{j} + z^3\hat{k}$ *and* S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$

19. Evaluate $\int_c yz\,dx + zx\,dy + xy\,dz$ where c is the curve $x^2 + y^2 = 1,\ z = 4$ using Stoke's theorem

V Answer any three questions: (3x5=15)

20. Find $curl\left(curl\left(\vec{F}\right)\right)$ *if* $\vec{F} = x^2y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$

21. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ & $x^2 + y^2 - z = 3$ *at* $(2,-1,2)$

22. Evaluate $\iiint_R \frac{dz\,dy\,dx}{(1+x+y+z)^3}$ where R is the region bounded by the planes $x = 0, y = 0, z = 0, x + y + z = 1$

23. Evaluate $\iiint_R dz\,dy\,dx$ where R is the region common to both the cylinders $x^2 + y^2 = a^2$ & $x^2 + z^2 = a^2$

24. Using Green's theorem find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

VI Answer any two questions: (2x5=10)

25. Find the work done in moving a particle under a force $\vec{F} = 2xy\,\hat{i} - 3x\,\hat{j} - 5z\,\hat{k}$ along the curve $x = t, y = t^2 + 1, z = 2t^2$ *from* $t = 0$ *to* $t = 1$

26. Find the x-coordinate of the centre of gravity of a mass of density $f(x, y) = 1$ in a region R, where R is a rectangular plate of dimension $2x4$ *ie* $0 \le x \le 2,\ 0 \le y \le 4$

27. Determine the y-coordinate of centre of gravity of the plane lamina of uniform surface density bounded by the upper half of the cardioid $r = a(1 + \cos\theta)$

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