BENGALURU CITY UNIVERSITY

SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7

MODEL PAPER – 1

Time: 3 hrs **Max marks: 70**

I Answer any five questions: **(5x2=10)**

1. Prove that the set $S = \{(3,2,-1),(0,4,5),(6,4,-2)\}$ is linearly dependent in $V_3(R)$
2. Define basis and dimension of a finite dimensional vector space.
3. Write scale factors in cylindrical co-ordinates.
4. Prove that in spherical co-ordinates system $\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi$.
5. Verify the integrability condition for $y^2dx + (x+z)^2dy + y^2dz = 0$.
6. Solve $\frac{dx}{z^2y} = \frac{dy}{z^2x} = \frac{dz}{xy^2}$.
7. Form the partial differential equation by eliminating arbitrary constant from $x^2 + y^2 = (z-c)^2 \tan^2 \alpha$, where 'c' and '$\alpha$' are arbitrary constants.
8. Solve $2p + 2q = 1$

II Answer any three questions: **(3x5=15)**

1. State & prove the necessary & sufficient condition for a non-empty subset W of a vector space V (F) to be a sub space of V.
2. Find the dimension and basis of the subspace spanned by the vectors $\{(2,4,2),(1,-1,0),(1,2,1),(0,3,1)\}$ of $V_3(R)$.
3. Find the matrix of the linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (x+y, 2z-x)$ relative to the bases $B_1 = \{(1,0,-1),(1,1,1),(1,0,0)\}$ and $B_2 = \{(0,1),(1,0)\}$
4. Let $T : V_3(R) \to V_3(R)$ be a linear transformation such that $T(1,0,0) = (1,0,2), T(0,1,0) = (1,1,0), T(0,0,1) = (1,-1,0)$. Find the range space, null space, rank, nullity and hence verify rank-nullity theorem.
5. Find all the eigen values & corresponding eigen vectors of the linear transformation,
   $T : V_2(R) \to V_2(R)$ defined by $T(x,y) = (x+4y, 2x+3y)$.

III Answer any three questions: **(3x5=15)**

1. Show that cylindrical co-ordinate system is orthogonal curvilinear co-ordinate system.
2. Express the vector $\vec{f} = yz\,\hat{\imath} + 2x\hat{\jmath} + y\,\hat{k}$ in cylindrical co-ordinates & find f_ρ, f_ϕ, f_z.

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Bengaluru City University
Central College Campus
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3. Express $\vec{f} = z\hat{\imath} - 2x\hat{\jmath} + y\hat{k}$ in spherical polar co-ordinates and hence find f_r, f_θ, f_ϕ.
4. Solve $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the condition
   (i) $u(0,t) = 0,\ \ u(1,t) = 0\ \forall\ t$
   (ii) $u(x,0) = x - x^2, 0 \le x \le 1$
5. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{\ell}\right)$, if it is released from rest from this position, find the displacement $y(x,t)$

IV Answer any three questions: (3x5=15)

1. Verify the condition of integrability and solve $2yz\,dx + zx\,dy - xy\,(1+z)dz = 0$
2. Solve: $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$
3. Solve: $x(1+y)p = y(1+x)q$
4. Find the complete integral of $px + qy = pq$ by Charpit's method
5. Solve $(D^2 - DD^1 - 2D^{12})\,z = (y-1)e^x$

V Answer any three questions: (3x5=15)

1. Find the equation of a set of curves intersecting the ellipsoids $2x^2 + 2y^2 + z^2 = a$ at right angles
2. Obtain the system of curves lying on the system of surfaces $xz = c$ and satisfying the differential equation $yzdx + z^2dy + y(z+x)dz = 0$
3. Evaluate $\iiint (x^2 + y^2 + z^2)\,dx\,dy\,dz$ where V is the sphere having centre at the region & radius equal to 'a' by changing the variable to spherical polar co-ordinates.
4. Obtain the solution to one dimensional heat equation using fourier series
5. An insulated rod of length l has its ends A and B maintained at 0^0 C and 100^0 Crespectively until steady state conditions prevail. If B is suddenly reduced to 0^0 C and maintained at 0^0 C. Find the temperature at a distance x from A at time t

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Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru 560 001

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SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7

MODEL PAPER – 2

Time: 3 hrs **Max marks: 70**

I Answer any five questions: **(5x2=10)**

1. Prove that in any vector space V over a field $F, c.0 = 0, \forall c \in F$ & 0 being the zero vector of V
2. Find the eigen values and corresponding eigen vectors of the identity linear transformation
3. Write scale factors in spherical co-ordinates
4. In cylindrical co-ordinate system prove that $\hat{e}_\phi . \hat{e}_z = 0$
5. Verify the integrability condition for $(y+z)dx+(x+z)dy+(x+y)dz=0$
6. Solve $\frac{dx}{y^2}=\frac{dy}{xz}=\frac{dz}{xy}$
7. Form the partial differential equation by eliminating the arbitrary function f from $z = f(x^2 - y^2)$
8. Solve $\sqrt{p}+\sqrt{q}=1$

II Answer any three questions: **(3x5=15)**

9. Show that $V = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} / x, y \in R \right\}$ is a vector space over R
10. Find the basis and dimension of the subspace spanned by $(1,-2,3),(1,-3,4),(-1,1,-2)$ of the vector space $V_3(R)$
11. Find the matrix of linear transformation $T: R^2 \to R^2$ defined by $T(x,y)=(x+4y, 2x-3y)$ relative to the bases $B_1 = \{(1,0),(0,1)\}, B_2 = \{(1,3),(2,5)\}$
12. If $T: U \to V$ be a linear transformation then prove that $(i) R(T)$ is a subspace of V $(ii) T$ is one-one if and only if $N(T)=\{0\}$
13. State and prove rank-nullity theorem

III Answer any three questions: **(3x5=15)**

14. Express the vector $\vec{f} = xz\hat{i} - 2y\hat{j} + y^2\hat{k}$ in spherical co-ordinates and find f_r, f_θ, f_ϕ
15. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z

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Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

16. Show that spherical co-ordinate system is orthogonal curvilinear co-ordinate system

17. Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

$(i)\, u(0,t) = 0,\ u(l,t) = 0,\ t \geq 0 \quad (ii)\, u(x,0) = \frac{100x}{l},\ 0 \leq x \leq l$

18. Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ $given\ u(0,t) = 0,\ u(l,t) = 0,\ u(x,0) = a\sin\left(\frac{\pi x}{l}\right) and \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

IV Answer any three questions: **(3x5=15)**

19. Verify the condition of integrability and solve

$3x^2dx + 3y^2dy - \left(x^3 + y^3 + e^{2z}\right)dz = 0$

20. Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$

21. Solve $xp + yq = z$

22. Solve $p^3 + q^3 = 27z$

23. Solve $\left(D^2 - 2DD' + D'^2\right)z = e^{x+2y} + x^3$

V Answer any 3 questions: **(3x5=15)**

24. Find a linear transformation $T: R^2 \to R^2$ such that

$T(1,0) = (1,1)\ \&\ T(0,1) = (-1,2)$

Prove that T maps the square with vertices $(0,0),(1,0),(1,1)\, and\, (0,1)$ into parallelogram

25. Find the family of surfaces $x^2 + y^2 + 2z^2 = c$

26. Find the curves which satisfy the differential equation

$ydx + zdy - ydy + xdz = 0$ and which lie on the plane $2x - y - z = 1$

27. Express the velocity 'v' and acceleration 'a' of a particle in cylindrical co-ordinates

28. A rod of length 'l' with insulated sides is initially at a uniform temperature 'u'. Its ends are suddenly cooled at $0^0 C$ and are kept at that temperature. Prove that the temperature function $u(x,t)$ is given by

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) e^{\frac{-c^2\pi^2 n^2 t}{l^2}}$$

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Chairman
Department of Mathematics
Bengaluru City University
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SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7

MODEL PAPER – 3

Time: 3 hrs **Max marks: 70**

I Answer any five questions: **(5x2=10)**

1. Show that the subset $W = \{(x, y, z) / x, y, z \textit{ are rational numbers}\}$ is not a subspace of $V_3(R)$
2. Find the matrix of linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x, -y)$ with respect to the standard bases
3. In cylindrical co-ordinate system prove that $\hat{e}_\rho . \hat{e}_\phi = 0$
4. Write the relation between the cartesian co-ordinates and spherical co-ordinates of a point
5. Verify the integrability condition for $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$
6. Solve $\frac{xdx}{y^2 z} = \frac{dy}{zx} = \frac{dz}{y^2}$
7. Solve $(px + qy - z)^2 = 1 + p^2 + q^2$
8. Solve $(D^2 - 4DD' + 4D'^2)z = 0$

II Answer any three questions: **(3x5=15)**

9. Find the dimension and basis of the subspace spanned by $\{(2,-3,1),(3,0,1),(0,2,1),(1,1,1)\}$ in $V_3(R)$
10. In an n-dimensional vectorspace $V(F)$ prove that (i) *any* $(n+1)$ vectors of V are linearly dependent. (ii) no set of $(n-1)$ vectors can span V
11. Given the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}$ Find the linear transformation $T : V_2(R) \to V_3(R)$ relative to the bases $B_1 = \{(1,1),(-1,1)\}$, $B_2 = \{(1,1,1),(1,-1,1),(0,0,1)\}$
12. Find the range space , null space , rank , nullity and hence verify rank-nullity theorem for $T : V_3(R) \to V_2(R)$ defined by $T(x, y, z) = (y - x\ ,\ y - z)$
13. Show that the set of all eigen vectors associated with the eigen value λ of a linear transformation T together with zero vector is a subspace of the vectorspace

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

III Answer any three questions: **(3x5=15)**

14. Show that the spherical co-ordinate system is an orthogonal curvilinear co-ordinate system

15. Express $\vec{f} = 3x\hat{i} - 2yz\hat{j} + x^2 z\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z

16. Express $\vec{f} = x\hat{i} + y\hat{j} + z\hat{k}$ in spherical co-ordinate system and find f_r, f_θ, f_ϕ

17. Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ given $u(0,t) = 0, u(1,t) = 0\ ,\forall t$

$u(x,0) = x^2 - x\ ,\ 0 \le x \le 1$

18. Solve $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ given $u(0,t) = 0\ ,\ u(l,t) = 0$

$u(x,0) = k(lx - x^2),\ \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

IV Answer any three questions: **(3x5=15)**

19. Verify the condition for integrability and solve

$z^2dx + (z^2 - 2yz)dy + (2y^2 - yz - zx)dz = 0$

20. Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$

21. Form the partial differential equation by eliminating arbitrary function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$

22. Solve $x^2p^2 + y^2q^2 = z^2$ by taking $u = \log x,\ v = \log y,\ w = \log z$

23. Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by using Charpit's method

V Answer any three questions: **(3x5=15)**

24. Find the family of curves orthogonal to the family of surfaces

$x^2 + 2y^2 + 4z^2 = c$

25. The vibration of an elastic string is governed by the pde $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ The length of the string is π and ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$ Find the deflection $u(x,t)$ of the vibrating string for $t > 0$

26. Express each of the following loci in spherical co-ordinates

(i) The sphere $x^2 + y^2 + z^2 = 9$

(ii) The cone $z^2 = 3(x^2 + y^2)$

(iii) The paraboloid $z = x^2 + y^2$

(iv) The plane $z = 0$

(v) The plane $y = x$

27. Reduce the equation $r + 2s + t = 0$ to canonical form

28. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2\frac{\partial^2 z}{\partial y^2}$ to canonical form

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001