

BENGALURU CITY UNIVERSITY

SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7

MODEL PAPER – 1

Time: 3 hrs

Max marks: 70

I Answer any five questions:

(5x2=10)

1. Prove that the set $S = \{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is linearly dependent in $V_3(\mathbb{R})$
2. Define basis and dimension of a finite dimensional vector space.
3. Write scale factors in cylindrical co-ordinates.
4. Prove that in spherical co-ordinates system $\widehat{e}_r \times \widehat{e}_\theta = \widehat{e}_\phi$.
5. Verify the integrability condition for $y^2 dx + (x+z)^2 dy + y^2 dz = 0$.
6. Solve $\frac{dx}{z^2 y} = \frac{dy}{z^2 x} = \frac{dz}{xy^2}$.
7. Form the partial differential equation by eliminating arbitrary constant from $x^2 + y^2 = (z - c)^2 \tan^2 \alpha$, where 'c' and ' α ' are arbitrary constants.
8. Solve $2p + 2q = 1$

II Answer any three questions:

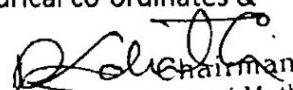
(3x5=15)

1. State & prove the necessary & sufficient condition for a non-empty subset W of a vector space $V(F)$ to be a sub space of V .
2. Find the dimension and basis of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ of $V_3(\mathbb{R})$.
3. Find the matrix of the linear transformation $T : V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x, y, z) = (x + y, 2z - x)$ relative to the bases $B_1 = \{(1, 0, -1), (1, 1, 1), (1, 0, 0)\}$ and $B_2 = \{(0, 1), (1, 0)\}$
4. Let $T : V_3(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ be a linear transformation such that $T(1, 0, 0) = (1, 0, 2), T(0, 1, 0) = (1, 1, 0), T(0, 0, 1) = (1, -1, 0)$. Find the range space, null space, rank, nullity and hence verify rank-nullity theorem.
5. Find all the eigen values & corresponding eigen vectors of the linear transformation,
 $T : V_2(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x, y) = (x + 4y, 2x + 3y)$.

III Answer any three questions:

(3x5=15)

1. Show that cylindrical co-ordinate system is orthogonal curvilinear co-ordinate system.
2. Express the vector $\vec{f} = yz \hat{i} + 2x \hat{j} + y \hat{k}$ in cylindrical co-ordinates & find f_ρ, f_ϕ, f_z .


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3. Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical polar co-ordinates and hence find f_r, f_θ, f_ϕ .
4. Solve $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the condition
 - (i) $u(0, t) = 0, u(1, t) = 0 \forall t$
 - (ii) $u(x, 0) = x - x^2, 0 \leq x \leq 1$
5. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$, if it is released from rest from this position, find the displacement $y(x, t)$

IV Answer any three questions:

(3x5=15)

1. Verify the condition of integrability and solve $2yz dx + zx dy - xy(1+z)dz = 0$
2. Solve: $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$
3. Solve: $x(1+y)p = y(1+x)q$
4. Find the complete integral of $px + qy = pq$ by Charpit's method
5. Solve $(D^2 - DD^1 - 2D^{12})z = (y-1)e^x$

V Answer any three questions:

(3x5=15)

1. Find the equation of a set of curves intersecting the ellipsoids $2x^2 + 2y^2 + z^2 = a$ at right angles
2. Obtain the system of curves lying on the system of surfaces $xz = c$ and satisfying the differential equation $yzdx + z^2dy + y(z+x)dz = 0$
3. Evaluate $\iiint (x^2 + y^2 + z^2) dx dy dz$ where V is the sphere having centre at the origin & radius equal to 'a' by changing the variable to spherical polar co-ordinates.
4. Obtain the solution to one dimensional heat equation using fourier series
5. An insulated rod of length l has its ends A and B maintained at 0° C and 100° C respectively until steady state conditions prevail. If B is suddenly reduced to 0° C and maintained at 0° C. Find the temperature at a distance x from A at time t



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SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7

MODEL PAPER – 2

Time: 3 hrs

Max marks: 70

I Answer any five questions:

(5x2=10)

1. Prove that in any vector space V over a field F , $c \cdot 0 = 0, \forall c \in F$ & 0 being the zero vector of V
2. Find the eigen values and corresponding eigen vectors of the identity linear transformation
3. Write scale factors in spherical co-ordinates
4. In cylindrical co-ordinate system prove that $\hat{e}_\phi \cdot \hat{e}_z = 0$
5. Verify the integrability condition for $(y+z)dx + (x+z)dy + (x+y)dz = 0$
6. Solve $\frac{dx}{y^2} = \frac{dy}{xz} = \frac{dz}{xy}$
7. Form the partial differential equation by eliminating the arbitrary function f from $z = f(x^2 - y^2)$
8. Solve $\sqrt{p} + \sqrt{q} = 1$

II Answer any three questions:

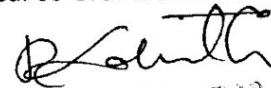
(3x5=15)

9. Show that $V = \left\{ \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} / x, y \in R \right\}$ is a vector space over R
10. Find the basis and dimension of the subspace spanned by $(1, -2, 3), (1, -3, 4), (-1, 1, -2)$ of the vector space $V_3(R)$
11. Find the matrix of linear transformation $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to the bases $B_1 = \{(1, 0), (0, 1)\}, B_2 = \{(1, 3), (2, 5)\}$
12. If $T: U \rightarrow V$ be a linear transformation then prove that
(i) $R(T)$ is a subspace of V (ii) T is one-one if and only if $N(T) = \{0\}$
13. State and prove rank-nullity theorem

III Answer any three questions:

(3x5=15)

14. Express the vector $\vec{f} = xz\hat{i} - 2y\hat{j} + y^2\hat{k}$ in spherical co-ordinates and find f_r, f_θ, f_ϕ
15. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z


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16. Show that spherical co-ordinate system is orthogonal curvilinear co-ordinate system

17. Solve $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

$$(i) u(0, t) = 0, u(l, t) = 0, t \geq 0 \quad (ii) u(x, 0) = \frac{100x}{l}, 0 \leq x \leq l$$

18. Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0, u(l, t) = 0, u(x, 0) = a \sin\left(\frac{\pi x}{l}\right)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$

IV Answer any three questions:

(3x5=15)

19. Verify the condition of integrability and solve

$$3x^2 dx + 3y^2 dy - (x^3 + y^3 + e^{2z}) dz = 0$$

20. Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$

21. Solve $xp + yq = z$

22. Solve $p^3 + q^3 = 27z$

23. Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3$

V Answer any 3 questions:

(3x5=15)

24. Find a linear transformation $T: R^2 \rightarrow R^2$ such that

$$T(1, 0) = (1, 1) \text{ \& } T(0, 1) = (-1, 2)$$

Prove that T maps the square with vertices $(0, 0), (1, 0), (1, 1)$ and $(0, 1)$ into parallelogram

25. Find the family of surfaces $x^2 + y^2 + 2z^2 = c$

26. Find the curves which satisfy the differential equation

$$ydx + zdy - ydz + xdz = 0 \text{ and which lie on the plane } 2x - y - z = 1$$

27. Express the velocity 'v' and acceleration 'a' of a particle in cylindrical co-ordinates

28. A rod of length 'l' with insulated sides is initially at a uniform temperature 'u'. Its ends are suddenly cooled at $0^\circ C$ and are kept at that temperature.

Prove that the temperature function $u(x, t)$ is given by

$$u(x, t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) e^{-\frac{c^2 \pi^2 n^2 t}{l^2}}$$



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SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 7
MODEL PAPER – 3

Time: 3 hrs

Max marks: 70

I Answer any five questions:

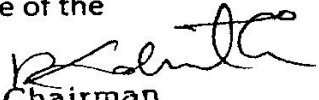
(5x2=10)

1. Show that the subset $W = \{(x, y, z) / x, y, z \text{ are rational numbers}\}$ is not a subspace of $V_3(\mathbb{R})$
2. Find the matrix of linear transformation $T: V_2(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x, y) = (x, -y)$ with respect to the standard bases
3. In cylindrical co-ordinate system prove that $\hat{e}_\rho \cdot \hat{e}_\phi = 0$
4. Write the relation between the cartesian co-ordinates and spherical co-ordinates of a point
5. Verify the integrability condition for $(2x^2 + 2xy + 2xz^2 + 1)dx + dy + 2zdz = 0$
6. Solve $\frac{xdx}{y^2z} = \frac{dy}{zx} = \frac{dz}{y^2}$
7. Solve $(px + qy + rz)^2 = 1 + p^2 + q^2$
8. Solve $(D^2 - 4DD' + 4D'^2)z = 0$

II Answer any three questions:

(3x5=15)

9. Find the dimension and basis of the subspace spanned by $\{(2, -3, 1), (3, 0, 1), (0, 2, 1), (1, 1, 1)\}$ in $V_3(\mathbb{R})$
10. In an n -dimensional vectorspace $V(F)$ prove that (i) any $(n+1)$ vectors of V are linearly dependent. (ii) no set of $(n-1)$ vectors can span V
11. Given the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 3 \end{pmatrix}$ Find the linear transformation $T: V_2(\mathbb{R}) \rightarrow V_3(\mathbb{R})$ relative to the bases $B_1 = \{(1, 1), (-1, 1)\}$, $B_2 = \{(1, 1, 1), (1, -1, 1), (0, 0, 1)\}$
12. Find the range space, null space, rank, nullity and hence verify rank-nullity theorem for $T: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})$ defined by $T(x, y, z) = (y - x, y - z)$
13. Show that the set of all eigen vectors associated with the eigen value λ of a linear transformation T together with zero vector is a subspace of the vectorspace


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III Answer any three questions:

(3x5=15)

14. Show that the spherical co-ordinate system is an orthogonal curvilinear co-ordinate system

15. Express $\vec{f} = 3x\hat{i} - 2yz\hat{j} + x^2z\hat{k}$ in cylindrical co-ordinates and find f_ρ, f_ϕ, f_z

16. Express $\vec{f} = x\hat{i} + y\hat{j} + z\hat{k}$ in spherical co-ordinate system and find f_r, f_θ, f_ϕ

17. Solve $\frac{\partial u}{\partial t} = 16 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0, u(1, t) = 0, \forall t$

$$u(x, 0) = x^2 - x, 0 \leq x \leq 1$$

18. Solve $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0, u(l, t) = 0$

$$u(x, 0) = k(lx - x^2), \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$$

IV Answer any three questions:

(3x5=15)

19. Verify the condition for integrability and solve

$$z^2 dx + (z^2 - 2yz) dy + (2y^2 - yz - zx) dz = 0$$

20. Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$

21. Form the partial differential equation by eliminating arbitrary function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$

22. Solve $x^2 p^2 + y^2 q^2 = z^2$ by taking $u = \log x, v = \log y, w = \log z$

23. Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by using Charpit's method

V Answer any three questions:

(3x5=15)

24. Find the family of curves orthogonal to the family of surfaces

$$x^2 + 2y^2 + 4z^2 = c$$

25. The vibration of an elastic string is governed by the pde $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ The

length of the string is π and ends are fixed. The initial velocity is zero and

the initial deflection is $u(x, 0) = 2(\sin x + \sin 3x)$ Find the deflection $u(x, t)$ of

the vibrating string for $t > 0$

26. Express each of the following loci in spherical co-ordinates

(i) The sphere $x^2 + y^2 + z^2 = 9$

(ii) The cone $z^2 = 3(x^2 + y^2)$


(iii) The paraboloid $z = x^2 + y^2$

(iv) The plane $z = 0$

(v) The plane $y = x$

27. Reduce the equation $r + 2s + t = 0$ to canonical form

28. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form


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