BENGALURU CITY UNIVERSITY

SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 8

MODEL PAPER – 1

Time: 3 hrs **Max marks: 70**

I Answer any 5 questions: **(5x2=10)**

1. Show that $\arg\left(\frac{\bar{z}}{z}\right)=\frac{\pi}{2}$ represents a line through the origin.
2. Define continuity of $f(z)$ at the point $z=z_0$
3. Prove that $f(z)=e^z$ is an analytic function.
4. Show that $u=e^x\sin y+x^2-y^2$ is a harmonic function
5. Evaluate $\oint_c(\bar{z})^2\,dz$ around the circle $|z|=1$
6. Define a bilinear transformation
7. Find the real root of the equation $x^3-x-2=0$ in the interval $(1.5,2)$ upto 2 approximations by bisection method
8. State formula for Runge - Kutta method

II Answer any 3 questions: **(3x5=15)**

9. Find the locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$

10. State and prove necessary condition for a function $f(z)=u(x,y)+i\,v(x,y)$ to be analytic

11. Show that $f(z)=\cos z$ is analytic and hence prove that $f'(z)=-\sin z$

12. Prove that $u(x,y)=y^3-3x^2y$ is a harmonic function. Determine its harmonic conjugate

13. Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y-\sin y)$

III Answer any 3 questions: **(3x5=15)**

14. Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)dx+(2y-x)dy$ along $(i)\ y=x^2+1$ (ii) line joining $(0,1)\,\&\,(2,5)$ Int

15. State and prove Cauchy's Integral theorem

16. Evaluate $\int_c\frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}dz$ where c is the circle $|z|=3$

17. Discuss the transformation $w=\sin z$

18. Find the bilinear transformation which maps the points $z=1,i,-1$ into $w=2,i,-2$

Chairman

Department of Mathematics

Bengaluru City University

Central College Campus

Bengaluru 560 001

IV Answer any 3 questions: **(3x5=15)**

19. Using Newton- Raphson method find the real root of the equation $x^3+5x-11=0$ by performing 3 iterations only.
20. Solve the equations $10x+2y+z=9$, $x+10y-z=-22$, $-2x+3y+10z=22$ by Gauss-Seidel method
21. Find the largest eigen value of the matrix $\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ by power method
22. Solve $\frac{dy}{dx}=\frac{y-x}{y+x}$ *with* $y(0)=1$ *for* $x=0.1$ by Euler's method in five steps
23. By using Runge-Kutta method solve $\frac{dy}{dx}=3x+\frac{y}{2}$ *with* $y(0)=1$

    Compute $y(0.2)$ by taking $h=0.2$

V Answer any 3 questions: **(3x5=15)**

24. If $w=\phi+i\psi$ represents the complex potential for an electric field and $\psi=x^2-y^2+\frac{x}{x^2+y^2}$ Determine the potential function ϕ and complex potential function w
25. An electrostatic field in the xy-plane is given by the potential function $\phi=3x^2y-y^3$ Find the stream function ψ and also the complex potential function w
26. If the potential function is $\log(x^2+y^2)$ Find the flux function and complex potential function
27. The concentration of salt x in a homemade soap maker is given as a function of time by $\frac{dx}{dt}=37.5-3.5x$ At the initial time, $t=0$ the salt concentration in the tank is $50g/L$ Using Runge Kutta method with a step size of $h=1.5$ min What is the salt concentration after 1.5 mins
28. A polluted lake has an initial concentration of a bacteria of 10^7 parts/m^3 while the acceptable level is only 5×10^6 parts/m^3 The concentration of the bacteria will reduce as fresh water enters the lake. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by $\frac{dC}{dt}+0.06C=0$, $C(0)=10^7$ Using Euler's method and a step size of 3.5 weeks, find the concentration of the pollutant after 7 weeks.

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SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 8

MODEL PAPER – 2

Time: 3 hrs **Max marks: 70**

I Answer any 5 questions: **(5x2=10)**

1. Show that $\left|\frac{z-2}{z+2}\right| = 3$ represents a circle
2. Show that $\lim_{z\to 0}\left(\frac{xy}{x^2+y^2}\right)$ does not exist
3. Define harmonic function. Give an example
4. Verify that $u = x^2 - y^2$ & $v = 2xy$ are the real and imaginary parts of an analytic function
5. State Cauchy's inequality
6. Find the fixed points of the bilinear transformation $w = \frac{3z-4}{z-1}$
7. Write Newton-Raphson iterative formula
8. Using power method find the largest eigen value of $\begin{pmatrix} 4 & 5 \\ 1 & 2 \end{pmatrix}$. Do 3 steps only

II Answer any 3 questions: **(3x5=15)**

9. Find the locus of the point z satisfying $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$

10. Evaluate $\lim_{z\to 2e^{i\pi/4}}\left(\frac{z^2-4}{z^3+z+5}\right)$

11. Find the orthogonal trajectories of the families of curves $e^{-x}\cos y + xy = c$

12. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate

13. If $f(z) = u + iv$ is an analytic function then prove that

$$\left(\frac{\partial f(z)}{\partial x}\right)^2 + \left(\frac{\partial f(z)}{\partial y}\right)^2 = |f'(z)|^2$$

III Answer any 3 questions: **(3x5=15)**

14. Evaluate $\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3y - x)dy$ along the curve $x = 2t,\ y = t^2 + 3$

15. State and prove Cauchy's integral formula

16. Evaluate $\int_c \frac{e^{3z}}{(z+1)^2(z-2)}$ *where* $c : |z| = 3$

17. Discuss the transformation $w = e^z$

18. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = 0, i, \infty$ respectively

Chairman
Department of Mathematics
Bengaluru City University
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IV Answer any 3 questions: **(3x5=15)**

19. Using bisection method find the real root of $x^3 - 3x^2 + 1 = 0$ correct to 3 decimal places

20. Solve the equations $x + y + 54z = 110$, $27x + 6y - z = 85$, $6x + 15y + 2z = 7$ by using Gauss-Jacobi iteration method correct to 2 decimal places

21. Find the largest eigen value of the matrix $\begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by p ower method

22. Using Taylor series method find y *at* $x = 0.2$ correct to 3 decimal places given $\frac{dy}{dx} = x - y^2$ & $y(0) = 1$

23. Solve $\frac{dy}{dx} = x - y$ by Euler's modified method with $y(0) = 1$ *for* $x = 0.2$ correct to 3 decimal places taking $h = 0.1$

V Answer any 3 questions: **(3x5=15)**

24. In a two dimensional fluid flow, if $xy\left(x^2 - y^2\right)$ represents the stream function Find the corresponding velocity function and also complex potential function

25. Two concentric circular cylinders of radii r_1, r_2 $(r_1 < r_2)$ are kept at potentials ϕ_1 and ϕ_2 respectively. Using complex function $w = a\log z + c$ prove that the capacitance per unit length of the capacitor formed by them is $\frac{2\pi\lambda}{\log\left(\frac{r_2}{r_1}\right)}$ where λ is the dielectric constant of the medium

26. Show that $u = -wy, v = wx, w = 0$ represents a possible motion of inviscid fluid. Find the stream function and sketch stream lines.

27. The open loop response , that is , the speed of the motor to a voltage input of $20V$ assuming a system without damping is $20 = 0.02\frac{dw}{dt} + 0.06w$ If the initial speed is zero $(w(0) = 0)$ and using Euler's method what is the speed at $t = 0.8s$ Assume a step size of $h = 0.4s$

28. A polluted lake has an initial concentration of a bacteria of 10^7 parts/m^3 while the acceptable level is only 5×10^6 parts/m^3 The concentration of the bacteria will reduce as fresh water enters the lake. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by $\frac{dC}{dt} + 0.06C = 0$, $C(0) = 10^7$ Using Runge Kutta method find the concentration of the pollutant after 3.5 weeks. Take a step size of 3.5 weeks.

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BENGALURU CITY UNIVERSITY

SIXTH SEMESTER, B.Sc., MATHEMATICS/PAPER- 8

MODEL PAPER – 3

Time: 3 hrs **Max marks: 70**

I Answer any 5 questions: **(5x2=10)**

1. Find the locus of the point z satisfying $|z-i|\leq 3$
2. Evaluate $\lim\limits_{z\to -2i}\dfrac{(2z+3)(z-1)}{z^2-2z+4}$
3. Prove that $u=x^3-3xy^2$ is a harmonic function
4. Show that $f(z)=\sin x\cosh y+i\cos x\sinh y$ is analytic
5. Evaluate $\int\limits_0^{1+i}(\bar{z})^2\,dz$ *along* $y=x$
6. Define cross ratio of 4 points z_1, z_2, z_3, z_4
7. Find first approximation root of the equation $f(x)=x^3-x-1$ by Regula Falsi method
8. Find the square root of the number 45 using Newton-Raphson method

II Answer any 3 questions: **(3x5=15)**

9. Find the locus of the point z satisfying $\arg\left(\dfrac{z-1+i}{z+1}\right)=\dfrac{\pi}{4}$
10. Prove with usual notations $\dfrac{\partial u}{\partial r}=\dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$, $\dfrac{\partial u}{\partial \theta}=-r\dfrac{\partial v}{\partial r}$
11. Find the orthogonal trajectories of the family of curves $x^3y-xy^3=c$
12. Show that $f(z)=\log z$ is analytic and hence find $f'(z)$
13. If $f(z)=u+iv$ is an analytic function then prove that the curves $u(x,y)=c_1$ & $v(x,y)=c_2$ form two orthogonal families

III Answer any 3 questions: **(3x5=15)**

14. Evaluate $\int\limits_c (x+2y)dx+(4-2x)dy$ around the ellipse c defined by $x=4\cos\theta,\ y=3\sin\theta,\ 0\leq\theta\leq 2\pi$
15. State and prove fundamental theorem of algebra
16. Evaluate $\oint\limits_c \dfrac{z-4}{z(z^2+9)}dz$ where c is the circle $|z|=1$
17. Discuss the transformation $w=\sinh z$
18. Prove that the bilinear transformation preserves the cross ratio of 4 points

Chairman
Department of Mathematics
Bengaluru City University
Central College Campus
Bengaluru - 560 001

IV Answer any 3 questions: (3x5=15)

19. Find the real root of the equation $x^3 - 4x + 1 = 0$ by Regula Falsi method correct to 3 decimals

20. Using Newton- Raphson method show that the iterative formula for $\frac{1}{\sqrt{N}}$ is $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{Nx_n}\right)$

21. Solve by Gauss-Seidel method
$20x + 2y + 6z = 28$, $x + 20y + 9z = 23$, $2x - 7y - 20z = -57$

22. Using Taylor series method solve $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ find $y(0.1)$ correct to 3 decimals taking upto 4[th] degree term

23. Using Runge-Kutta method solve $\frac{dy}{dx} = \frac{1}{x+y}$, $y(0.4) = 1$ *at* $x = 0.5$ correct to 3 decimals

V Answer any 3 questions: (3x5=15)

24. A two dimensional flow field is given by $\psi = xy$ Show that the flow is irrotational. Also find the velocity potential. Find the stream lines and potential lines.

25. Expand $\frac{1}{z+1}$ about $z = 1$ in Taylor's series

26. Find the first four terms of the Taylor series expansion of complex variable function $f(z)$ about $z = 2$ where $f(z) = \frac{z+1}{(z-3)(z-4)}$

27. Solve $\frac{dy}{dx} + 2y = 1.3e^{-x}$, $y(0) = 5$ *with* $h = 0.1$ find $y(0.1)$ using Runge Kutta method

28. The concentration of salt x in a homemade soap maker is given as a function of time by
$\frac{dx}{dt} = 37.5 - 3.5x$ At the initial time $t = 0$ the salt concentration in the tank is $50g/L$ Using Euler's method and a step size of $h = 1.5\text{min}$ what is the salt concentration after 3 min

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Chairman
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Bengaluru City University
Central College Campus
Bengaluru 560 001