KLE Society's

S. NIJALINGAPPA COLLEGE

IInd Block, Rajajinagar, Bengaluru - 560 010.
Re-accredited by NAAC at 'A+' grade with 3.53 CGPA
College with UGC STRIDE Component-I

Proceedings

IQAC initiated UGC - STRIDE sponsored
Two Days National Level Webinar on

"MATHEMATICAL MODELLING IN TRANS - DISCIPLINARY RESEARCH ON BASIC SCIENCE"

AUGUST
26th & 27th 2020

ISBN :
978-93-5437-830-0

ISBN: 978-93-5437-830-0

K.L.E SOCIETY's
S. NIJALINGAPPA COLLEGE
Reaccredited by NAAC at " A^+ " grade with 3.53 CGPA
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II Block, Rajajinagar Bangalore - 10

Title: Proceedings of IQAC initiated UGC STRIDE sponsored **"National Webinar On Mathematical Modelling In Trans-Disciplinary Research On Basic Science"**

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Published By: Self Published
Publisher's Address: Dr. Arunkumar B. Sonappanavar, Principal, KLE Society's S. Nijalingappa College, 2nd block, Rajajinagar, Bengaluru, Karnataka- 560 010
Email: klesncmaths1963@gmail.com **Website:** http://www.klesnc.org **Tel:** 080-23526055
Publisher's Details: Dr. Arunkumar B. Sonappanavar, Principal, KLE Society's S. Nijalingappa College, 2nd block, Rajajinagar, Bengaluru, Karnataka- 560 010
Email: klesncmaths1963@gmail.com **Mobile:** +91 9483946362
Printer's Details:
K.L.E Society's S. Nijalingappa College II Block, Rajajinagar Bangalore-10, Karnataka
Edition details: I (First Edition), 2020
ISBN: 978-93-5437-830-0.

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Re-accredited by NAAC at 'A+' grade with 3.53 CGPA
College with UGC STRIDE Component – I

PROCEEDINGS

UGC – STRIDE Sponsored Two Day National Webinar Organised by Department of Mathematics

ON

"Mathematical Modelling in Trans-Disciplinary Research on Basic Science"

2020-21

ISBN: 978-93-5437-830-0

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About Society

KLE Society is one of the largest educational organisations in South Asia, which was established in 1916 by seven dedicated and selfless teachers who are referred to as "Saptha rishis". It has been transformed into a veritable movement in providing quality education over the past 100 years. The legacy of Society and its core values are being led by our Chairman Dr Prabhakar B. Kore, MP since 1983. The KLE family now encompasses over 16,000 staff serving in 256 Institutions catering to the needs of more than 1.25 lakh students. The Society has one of the rarest distinctions of being conferred with two Deemed Universities status-one in Medical Education and the other in Technical Education. In 2016, the Society celebrated its Centenary. Honourable Prime Minister Shri Narendra Modi graced the occasion. He acknowledged the sacrifice and yeoman service of "Saptharishis" and the Management.

About Our College

S. Nijalingappa College celebrated its Golden jubilee in the year 2013. It is one of the premier higher educational institutions managed by the KLE Society. Since its inception in 1963, the college has carved a niche for its quality education. The College is re-accredited at "A+" level by NAAC (3rd Cycle) with CGPA of 3.53 on 4.0 scale and recognized as "College with Potential for Excellence"– II Phase by UGC. The college sprawls over 4.92 acres in the heart of the city with the State-of-the-Art infrastructure. It offers a wide variety of programmes at under graduate and postgraduate levels with a judicious focus on traditional and professional domains.

About the Department

The Department of Mathematics was established in 1963 with vision "**Pursue value based and quality education in mathematics"** presently one of the pioneering departments of the institution that offers Post Graduate and Under Graduate courses.

Objectives of the Department

- To enhance the students logical, reasoning, analytical and problem solving skills.
- Training the students to face the challenging trends of the day.
- Inculcating aptitude for research in Mathematics.

The Department constitutes student projects, guest seminars, student seminars, etc., an integral part of the student activities. Faculty members are attended International/National/State Seminars/Workshop/Conference and presented the research papers. Faculty members are also published their research papers in reputed International journals.

Objective of the Seminar

Recent advances in theory and applications of mathematics have contributed much to the successful handling of certain problems in biology, physics, economics, engineering, computer networks and telecommunication. The overall aim of this seminar is to bring together the latest or innovative knowledge and advances in mathematics for handling complex systems, which may depend largely on methods from mathematical analysis, artificial intelligence, statistics, and engineering and telecommunication. The seminar should provide solutions by mathematical modelling, analysis, and applications of real-world complex systems. The topics in these seminars contain some new, novel, and innovative techniques and ideas that may stimulate further research in every branch of Basic Science and applications of mathematics for the complex systems. Mathematical analysis simplifies many physical problem to a great extent.

Theme:

Mathematical Modelling in Trans-Disciplinary Research on Basic Science

SubThemes:

- General topology
- Ordinary and Partial differential Equations
- Differential Geometry
- Graph Theory
- Mathematical Modelling
- Statistical Tool In Social Sciences.
- Fourier Series and Transforms
- Value Distribution Theory
- Thermodynamics
- Fibonacci Sequence
- Mathematics for Day to Day Life

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Academicians, Research Scholars, UG and PG students.

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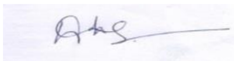
Schedule for the TWO DAY NATIONAL WEBINAR			
Day 1: 26th August, 2020			
Time	**Session**	**Topic**	**Resource Person**
10.30 – 12.00 pm	01		**Dr S. Sundar** DAAD Research Ambassador Head, Department of Mathematics IIT Madras, Chennai
12:00 – 1:30 pm	02	Differential Geometry	**Dr Nagaraja H G** Professor Department of Mathematics Bangalore University
Day 2: 27th August, 2020			
10.30 – 12.00 pm	03		**Dr S. Balaji** Associate Professor Department of Biotechnology Manipal Institute of Technology Manipal University, Manipal
12:00 – 1:30 pm		Paper Presentation	**Dr Nagaraja H G** Professor Department of Mathematics Bangalore University **Dr Girish M. Sajjanshettar** Associate Professor-Senior Scale Department of Mathematics MIT Manipal

Valedictory 27th August, 2020
Dr Girish M Sajjanshettar
Associate Professor-Senior Scale
Department of Mathematics
MIT Manipal

Presided by

Dr. Arunkumar B. Sonappanavar, Principal



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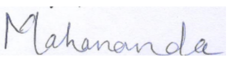

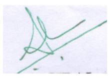

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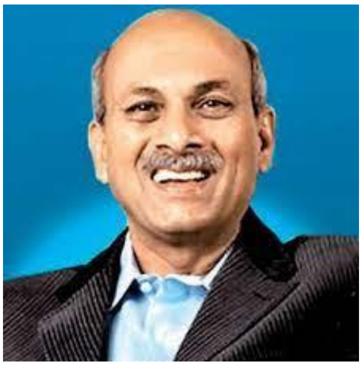
MESSAGE BY CHAIRMAN



Dr. Prabhakar B. Kore

Chairman
KLE Society, Belagavi
Karnataka

I am so delightful to know that IQAC of KLE Society's S. Nijalingappa College, Bengaluru is organising UGC-STRIDE Sponsored two day webinar on Mathematical Modelling in Trans-Disciplinary Research on Basic Science on 26th and 27th August, 2020 and has invited eminent resource personalities. I am confident that the webinar will enrich the knowledge of academicians and administrators of higher education institutions across the nation. Congratulations to the organisers and I wish the national webinar an epic magnificent success.

MESSAGE BY PRINCIPAL

Dr. Arunkumar B. Sonappanavar
Principal
KLE Society’s S Nijalingappa College
Rajajinagar, Bengaluru-560010.

It gives me immense pleasure to welcome all the eminent speakers and delegates to the IQAC initiated UGC-STRIDE sponsored Two-Days Webinar on “Mathematical Modelling in Trans-Disciplinary Research on Basic Science” 26th and 27th August, 2020 organized by Department of Mathematics, KLE Society’s S. Nijalingappa College, Bengaluru.

The webinar covers a keynote address followed by four technical sessions over the course of two days by eminent speakers’ research scholars and students across the country. I hope the two-day academic deliberations in the webinar will enlighten the faculty, researchers and students on recent strides in trans-disciplinary research and its applications for the benefit of humanity and encourage them to take up further research in these crucial fields of study. On this occasion, I extend a heartfelt welcome to all the delegates to KLE Society’s S. Nijalingappa College, Bengaluru. The college will bring out proceedings of the webinar. I congratulate organising committee members of webinar on conducting such an even to boost the knowledge of faculty, researchers and students.

Dr. Arunkumar B. Sonappanavar
Principal

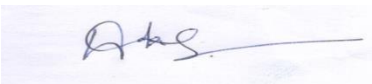
 ISBN: 978-93-5437-830-0

FOREWORD FROM CONVENER

Prof. Chandrashekharappa A. S.

Convener
KLE Society's S Nijalingappa College
Rajajinagar, Bengaluru-560010

I am immensely happy to organize the academic educational and participative IQAC initiated UGC-STRIDE Sponsored Two days National Level Webinar on "Mathematical Modelling in Trans-Disciplinary Research on Basic Science" at our college. It is another pleasurable, motivational and successful effort by Department of Mathematics to provide a platform to Scientists, Researchers, Academicians and students of Mathematics backgrounds to publish and exchange ideas, research and sharing their valuable experiences. The major objective of the events is to develop the interest of young researchers and students for undertaking studies and research. Earning and sharing knowledge is an external ocean. The progress is far away until we realise this truth. The abstracts and paper presented in the proceedings clearly show the multi-faced and inter-disciplinary nature of work. I thank the Management, Principal, Staff and Students of S. Nijalingappa College for the encouragement extended in organizing the Webinar. I also thank all those individuals and contributors who are involved directly or indirectly in making the Webinar a grand success.



Prof. Chandrashekharappa A. S.
Convener & HOD of Mathematics

AUTHOR INDEX

1. A STUDY ON GRAPH COLORING AND COLORING OF PLANAR GRAPHS

Bharathi S N, Assistant Professor of Mathematics, GFGC Kadugudi, Bangalore.

Graph theory, branch of mathematics concerned with networks of points connected by lines. The subject of graph theory had its beginnings in recreational math problems, but it has grown into a significant area of mathematical research, with applications in chemistry, operations research, social sciences, and computer science. The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler (1707-1782) solved the Königsberg bridge problem. The Konigsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island - but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially, he proved the first theorem in graph theory.

Definitions and Notations: A graph $\boldsymbol{G}$ is an ordered pair (V, E) where V is a finite non-empty set of vertices and E is a set of unordered pairs of distinct vertices of V. V is called the **vertex set** of G i.e., $V(G)$ and E is called the **Edge set** of G i.e., $E(G)$. The graph with p vertices and q edges is called a (p, q) graph. p is called the order of G and q , the size of G. A graph is called a trivial graph. Every pair $(u.v)$ of E is called an edge of G and their vertices u and v are said to be **adjacent**. The edge u, v is said to be incident to u and v. If two edges are incident to a vertex, then the two edges are said to be **adjacent edges.** The **complete** graph Kphas every pair of its vertices adjacent. A **bipartite graph** (bi – graph) G is a graph whose vertex set $V(G)$ can be partitioned into subsets V_1 and V_2 such that every edge in G joins a vertex in V_1, to a vertex in V_2 and $< V_1 >$, $< V_2 >$ are totally disconnected. If G contains every edge joining a vertex of V_1 to every vertex of V_2 , then G is called a **complete bipartite graph**. If $V_1$and V_2 have m and n vertices we write $G = K_{m,n}$. A star is complete bigraph $K_{1,n}$. A **walk** of a graph G is an alternating sequence of vertices and edges beginning and ending with vertices, such that, each edge is incident to the vertices proceeding and following it. Vertices with which a walk begins and ends are called **terminal vertices**. If the terminal vertices coincide then the walk is **closed walk**. A walk is a **trail** if all edges are distinct and a **path,** if all vertices are distinct. A closed path is called a **cycle**. A path with n vertices is denoted by $p_n$and cycle with$n$ vertices C_n. A graph is **acyclic** if it has no cycles. A tree is a connected acyclic graph.

GRAPH COLORING: In its simplest form, it is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color, called a **vertex coloring**. Similarly, an **edge coloring** assigns a color to each edge so that no two-incident edges share the same color, and a **face coloring** of a planar graph assigns a color to each face or region so that no two faces that share a boundary share the same color. Vertex coloring is the starting point of the subject, and other coloring problems can be transformed into a vertex version. The convention of using colors comes from graph drawings of graph colorings, where each node or edge is literally colored to indicate its mapping. In computer representations, it is more typical to use nonnegative integers, and in general, any mapping from the graph objects into a finite set can be used.

1. **Graph coloring** enjoys many practical applications as well as theoretical challenges. Beside the classical types of problems, different limitations can also be set on the graph, on the way a color is assigned, or even on the color itself. It has even reached popularity with the general public in the form of the popular number puzzle. Graph coloring is still a very active field of research.
2. Graph coloring has many applications in addition to its intrinsic interest.

* Sudoku is also a variation of Graph coloring problem where every cell represents a vertex. There is an edge between two vertices if they are in same row or same column or same block.

* If the vertices of a graph represent academic classes, and two vertices are adjacent if the corresponding classes have people in common, then a coloring of the vertices can be used to schedule class meetings. Here the colors would be schedule times, such as 8MWF, 9MWF, 11TTh, etc.

***** If the vertices of a graph represent radio stations, and two vertices are adjacent if the stations are close enough to interfere with each other, a coloring can be used to assign non-interfering frequencies to the stations

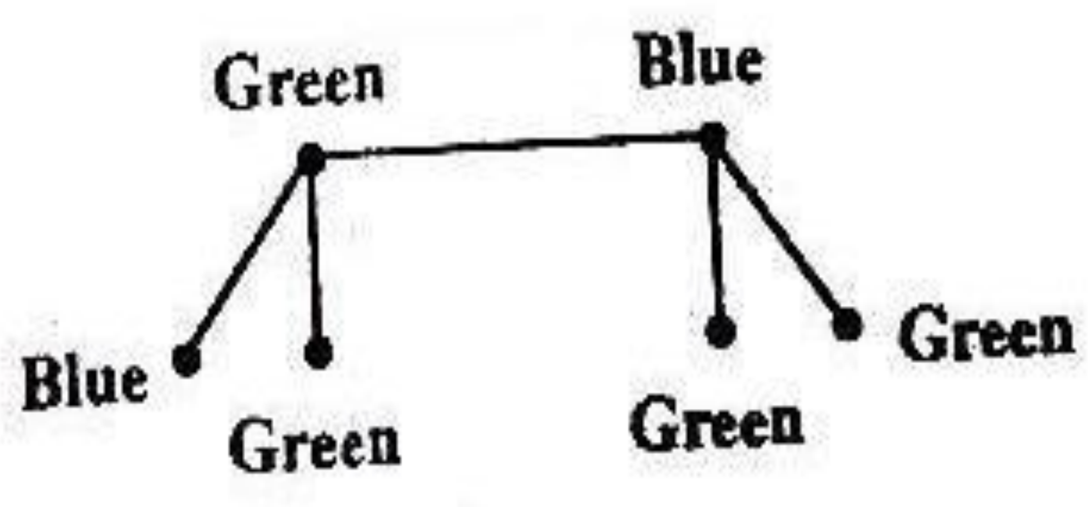
***** If the vertices of a graph represent traffic signals at an intersection, and two vertices are adjacent if the corresponding signals cannot be green at the same time, a coloring can be used to designate sets of signals than can be green at the same time.

*** Register Allocation:** In compiler optimization, register allocation is the process of assigning a large number of target program variables onto a small number of CPU registers. This problem is also a graph-coloring problem.

- We have seen several problems where it does not seem like graph theory should be useful.
    - But graphs are often very good at capturing the important stuff about a problem, so we can see it clearly.
- When drawing a map, we want to be able to distinguish different regions.
    - One of the ways that is done is by drawing them in different colours.
    - With the rule that regions that touch each other must be different colours: otherwise, we could not see the border.

Chromatic number:

The least number of colors needed to color the graph is called its **chromatic number.** It is denoted by the symbol $\Psi(G)$, where G is a graph. For example the chromatic number of a $K_n$of n vertices (a graph with an edge between every two vertices i.e., a complete graph with n vertices), is $\Psi(K_n) = n$. A graph that can be assigned a (proper) k-coloring is $\boldsymbol{k}$ **-colorable**, and it is $\boldsymbol{k}$**-chromatic** if its chromatic number is exactly k.



$\Psi(T) = 2$

PROPERTIES OF CHROMATIC NUMBERS

1. A graph, which is totally disconnected, has isolated vertices. No two vertices are adjacent. Therefore, the graph is 1-chromatic.
2. A graph containing one edge or more edges is at at least 2-chromatic.
3. A cycle with 3 vertices is 3-chromatic.
4. A graph, which is a cycle of $2n$ points, is 2-chromatic.
5. A complete graph with p-vertices is p-chromatic

6. A graph which is a cycle of $2n + 1$ points is 3-chromatic.
7. If $W_n$is a wheel, having one vertex at the centre and $n-1$ vertices along the circumference, $\Psi(W_n) = 4$ if n is even and $\Psi(W_n) = 3$ if n is odd integer.

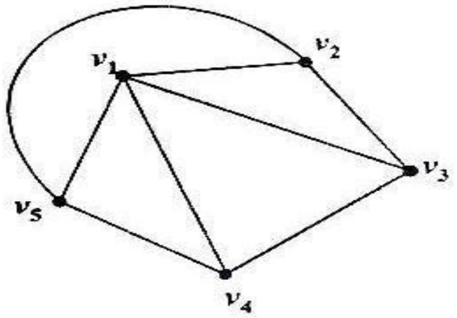
Chromatic Polynomial:

The **chromatic polynomial** counts the number of ways a graph can be colored using no more than a given number of colors. The value of the chromatic polynomial P_n (λ) of a graph with n vertices gives the number of ways of properly coloring the graph, using λ or fewer colors. Let C_i be the different ways of properly coloring G using exactly i different Colors. Since colors can be chosen out ofλcolors in $\binom{\lambda}{i}$or ${}^{\lambda}C_i$ different ways. There are ${}^{\lambda}C_i$ different ways of properly coloring G using exactly $i$colors out of λ colors. Since i can be any positive integer from 1 to n(it is not possible to use more than n colors on n vertices), the chromatic polynomial is a sum of these terms; that is,

$$P_n(\lambda) = \sum \boldsymbol{c_i} \binom{\lambda}{i} = \frac{c_1\lambda}{1!} + \frac{c_2\lambda(\lambda-1)}{2!} + \cdots + \frac{c_3\lambda(\lambda-1)(\lambda-2)\ldots(\lambda-\mathrm{n}+1)}{n!}$$

Each c_i has to be evaluated individually for the given graph. For example, graph with even one edge requires at least two colors for proper coloring, and therefore $c_1 = 0$A graph with n vertices and using n different colors can be properly colored in $n!$ ways; that is, $c_n = n!$

As an illustration, let us find the chromatic polynomial of the graph given in figure.



$$P_5(\lambda) = \frac{c_1\lambda}{1!} + \frac{c_2\lambda(\lambda-1)}{2!} + \frac{c_2\lambda(\lambda-1)(\lambda-2)}{3!} + \frac{c_3\lambda(\lambda-1)(\lambda-2)(\lambda-3)}{4!} \ldots + \frac{c_3\lambda(\lambda-1)(\lambda-2)(\lambda-3)(\lambda-4)}{5!}$$

Since the graph in figure has a triangle, it will require at least three different colors for proper coloring. Therefore,$c_1 = c_2 = 0$ and $c_5 = 5!$. Moreover, to evaluate c_3, suppose that we have three colors $x, y,$ and z. These three colors can be assigned properly to vertices v_1, v_2 and v_3 in $3! = 6$ different ways. Having done that, we have no more choices left, because vertex v_5 must have the same color as v_3 and v_4 must have the same color as v_2 . Therefore, $c_3 = 6$. Similarly, with four colors, v_1, v_2 and $v_3$can be properly colored in $4.6 = 24$ different ways. The fourth color can be assigned to $v_4$orv_5, thus providing two choices. The fifth vertex provides no additional choice. Therefore, $c_4 = 24.2 = 48$.

THEOREM: A graph of n vertices is complete graph if and only if its chromatic polynomial is, $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots\dots\dots (\lambda - n + 1)$.

Proof: With λcolors, there are λ different ways of coloring any selected vertex of a graph. A second vertex can be colored properly in exactly λ–1 ways, the third in λ–2 ways, the fourth in λ–3 ways…, and the nth $\lambda - n + 1$ ways if and only if every vertex is adjacent to every other. That is, if and only if the graph is complete.

BERGE AND ORE THEOREM:

In a k– chromatic graph, $\beta_o(G) \geq \frac{p}{k}$

(Same as proving $\beta_o(G) \geq \frac{p}{\Psi(G)}$)

Proof: $\Psi(G) = k$, because G is k – chromatic when we color with k colors, the vertices are partitioned into k – color classes, each color giving a class. Let $c_1, c_2, \dots c_k$ be k color classes and $P_1^|, P_2^|, \dots P_k^|$ be number of vertices in the color classes respectively. If $\beta_0$in the point independence number, β_0 gives the maximum number of non-adjacent points and they have the same color. Therefore β_0 = maximum of $(P_1, P_2, \dots, P_k)$

$$\therefore \beta_0 \geq P_1, \ \beta_0 \geq P_2 \dots \ \beta_0 \geq P_i \dots \beta_0 \geq P_k$$

where P_i is the maximum value of numbers $(P_1, P_2, \dots, P_k)$

$$\therefore \beta_o + \beta_o + \dots\dots\dots\dots\dots\dots\dots\dots + \beta_o \geq P_1 + P_{2\dots\dots\dots} + P_i + \dots\dots + P_k$$

$$k\beta_o \geq P_1 + P_2 + \dots\dots\dots\dots P_k$$

$\beta_o \geq$ p Where p is the total number of points of the graph

HARARY AND GADDUM THEOREM:

THEOREM: $\Psi(G) \leq p + 1 - \beta o$ Where p is the number points in G.

Proof: Let S be the maximal independent set containing βo points.

$$\therefore \Psi(G - S) = (K - 1)$$

$$= \Psi(b) - 1.$$

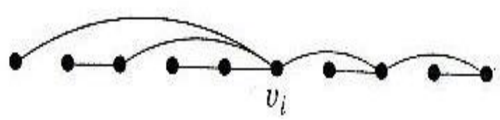
Maximum number of points in the set $G - S$ is $p - \beta o$. $\therefore$ Maximum number of colors that could be used. For coloring the set $G - S$ is $-\beta o$. Minimum colors are used for coloring, therefore $\Psi(G) - 1 \leq P - \beta o$. Therefore $\Psi(G) \leq P - \beta o + 1$.

BROOKS's THEOREM:

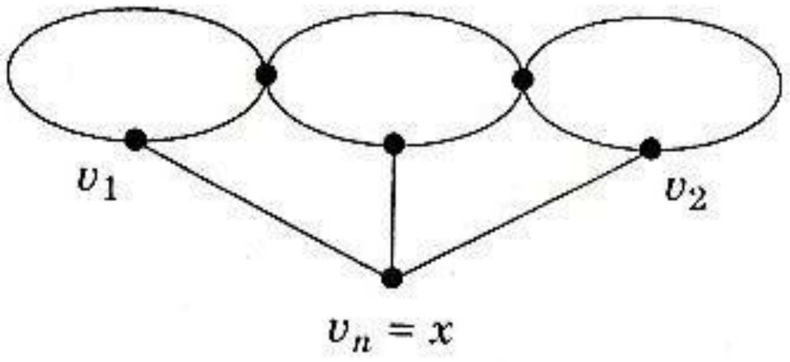
The bound $\Psi(G) \leq \Delta(G) + 1$ is tight for cliques and odd cycles. By choosing the vertex ordering more carefully, we can show that these are essentially the only graphs with $\Psi(G) > \Delta(G)$. This implies, for example, that the Peterson graph is 3 – colorable. To avoid unimportant complications, we phrase the statement only for connected graphs. It extends to all graphs because the chromatic number of a graph is the maximum chromatic number of its components.

BROOKS'S THEOREM [1941]: If a connected graph G is neither an odd cycle nor a complete graph, then $\Psi(G) \leq \Delta(G)$.

Proof: Suppose G is connected but it is not a clique or an odd cycle, and let $k = \Delta(G)$. We may assume that $k \geq 3$, since G is a clique when $k = 1$, and G is an odd cycle or is bipartite when $k = 2$. If G is not k-regular, choose v_n so that $d(v_n) < k$. Since G is connected, we can grow a spanning tree of G from v_n, assigning indices in decreasing order as we reach vertices. Each vertex other than v_n in the resulting ordering $v_1 \ldots v_n$ has higher indexed neighbor along the path to $v_n$in the tree. Hence each vertex has at most $k - 1$ lower indexed neighbors, and the greedy coloring uses at most k colors.



In the other case, G is k- Regular. If G has a cut–vertex x, let G' be a component of G-x together with its edges to x. The degree of x in G' is less than k, and we obtain a proper k- coloring of G' as above. By permuting the names of colors in each such sub graph, we can make the colorings, agree on x to complete a proper k – coloring of G. We may thus assume that G is 2- connected. Suppose G has an induced 3-vertex path, with vertices we call v_1, v_n, v_2 in order, such that $G - \{v_1, v_2\}$ is connected. We can then number the vertices of a spanning tree of $G - \{v_1, v_2\}$ using 3… n such that labels increase along paths to the root v_n. As before, each vertex before n as almost k-1 lower indexed neighbors. The greedy coloring also uses at most k-1 colors on neighbors of V_n, since V_1 &V_2 receive the same color.



Hence it enough to show that every 2-connected k-regular graph with $k \geq 3$ has three such vertices. Choose vertex x. If $K(G-x) \geq 2$, let V_1 be x and let V_2 be a vertex with distance two from x, which exists because G is regular and not a clique. If $K(G-x) = 1$, then x has a neighbor in every leaf block of G-x (since G has no cut-vertex). Neighbors V_1, V_2 of x in two such blocks are non-adjacent. Furthermore, $G-\{x, v_1, v_2\}$ is connected, since blocks have no cut-vertices. Now $k \geq 3$ implies that $G - \{ v_1, v_2 \}$ also is connected, and we let $V_n = x$. Hence the proof.

Graph coloring is closely related to the concept of an **independent set**.

Definition: A set S of vertices in a graph is independent if no two vertices of S are adjacent. If a graph is properly colored, the vertices that are assigned a particular color form an independent set. Given a graph G it is easy to find a proper coloring: give every vertex a different color. Clearly the interesting quantity is the minimum number of colors required for a coloring. It is also easy to find independent sets: just pick vertices that are mutually non-adjacent. A single vertex set, for example, is independent, and usually finding larger independent sets is easy. The interesting quantity is the maximum size of an independent set.

Planar Graphs: A graph is said to be a planar graph if there exists a geometric representation of G drawn on a plane such that no two edges intersect or cross each other. A graph that cannot on a plane without a cross over or intersection between the edges, is called a non-planar graph. The planar graph has faces (F), vertices (V) and edges (E). There is an Important relationship between V, E and F and this relation is $V - E + F = 2$, although known to Rene Descartes, was proved by Euler. Therefore this property is called Euler's theorem.

Results: 1) If G (p, q) is a connected planar graph where $p \geq 3$, then $q \leq 3p - 6$.

2) If G (p, q) is a connected planar graph, then it has at least one vertex of degree *five or less*.

Definitions:

Faces or Regions: A planar graph G, partitions the plane into many regions. These regions are also called faces. The bounded regions are called interior faces and unbounded region is called the exterior face.

Degree of a Region: A face is said to be incident with vertices and edges forming the boundary of the face. The degree of the face is the number of edges incident on the face.

Note: A tree (a connected acyclic graph) has only one face which is exterior and hence every tree with two or more vertices is 2-chromatic.

DUAL GRAPHS: From any plane graph G, we can form another plane graph called its "dual".

Definition: Suppose G is a plane graph. The *dual graph* G* of G is a plane graph having a vertex for each region in G. The edges of G* correspond to the edges of G as follows: if *e* is an edge of G that has region X on one side and region Y on the other side, then the corresponding dual edge $e^* \in E(G^*)$ is an edge joining the vertices x , y of G* that correspond to the faces X, Y of G.

Maximum planar graph: Maximal planar graph is a planar graph to which no further edge can be added without losing the planarity.

Outer planar graph: If all vertices of a graph belong to the same face, then the connected graph G is called outer planar graph.

Maximal outer planar graph: The outer planar graph is said to be maximal outer planar graph if no additional line can be brought to the graph without the loss of outer planarity.

Color problem of regions in a planar graph: The regions of a planar graph is properly colored if no two adjacent regions have the same color. Two regions are called adjacent regions if the regions have a common edge between them. If a graph is planar, we know that it has a dual. The coloring of vertices in a graph is the same as the coloring of regions is a dual planar graph and vice versa.

EULER'S FORMULA: Euler's Formula is the basic computational tool for planar graphs.

Theorem: (Euler [1758]): If a connected plane graph G has n vertices, e edges, and f faces, then $n - e + f = 2$.

Proof: Proof by induction on n. If n=1, then G is a″bouquet″ of loops. Each a closed curve in the embedding. If e=0, then we have one face, and the formula holds. Each added loop passes through a region and partitions it into two regions (by the Jordan Curve Theorem), so the formula holds for n=1 and any $e \geq 0$.

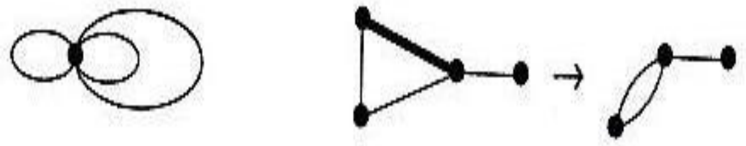


Diagram 3.1.8

Now suppose n (G)>1. Since G is connected, we can find an edge that is not a loop. When we contract such an edge, we obtain a plane graph G′. With n'vertices, e' edges, and f' faces. The contraction does not change the number of faces (we merely shortened boundaries), but it reduces the number of edges and vertices by one. Applying the induction hypothesis, we find $n–e+f=n'+1–(e'+1)+f=2$.

REMARK:

1) Euler's formula implies that all planar embedding of a connected graph G have the same number of faces. Thus, although the dual may depend on the embedding chosen for G, the number of vertices in the dual does not.
2) Contracting a non-loop edge of G has the effect of deleting an edge in G*. Similarly, deleting a non-cut edge of G has the effect of contracting an edge in G*, as two faces of G merge into a single face.
3) Euler's formula as stated fails for disconnected graphs. If a plane graph G has k components, then we can adjust Euler's formula by observing that adding k−1 edges to G will yield a connected plane graph without changing the components is $n–e+f$=k+1 (for example, consider a graph with *n* vertices and no edges).

Euler's formula has many applications, particularly for simple plane graphs, where all faces have length at least 3.

Theorem: If G is a simple planar graph with at least three vertices, then e (G) ≤ 3n (G)–6. If also G is triangle-free, then e (G) ≤ 2n (G)–4.

Proof: It suffices to consider connected graphs, since otherwise we could add edges. We can use Euler's formula to relate n(G) and e(G) if we can dispose of f. Proposition 7.1.11 provides an inequality between e and f .Every face boundary in a simple graph contains at least three edges (if n(G)≥ 3). If { f_i } is the sequence of face-lengths, this yields angle-free, then the faces have length at least four. In this case $2e = \Sigma f_i \geq 4f$, and we obtain e≤ 2n–4.

EXAMPLE: K_5 and $K_{3,3}$. Euler's formula incorporates the earlier geometric reasoning we used to show that $K_5$and $K_{3,3}$, are nonplanar. The non-planarity follows immediately from the edge bound. For K_5, we have e=10> 9 = 3n–6, and for the triangle-free $K_{3,3}$ we have e = 9>8 = 2n –4.

Characterization of Planar Graphs:

Which graphs embed in the plane? In the previous chapter we have shown that K_5 and$K_{3,3}$ do not. In a natural sense, these are the critical graphs and yield a characterization of planarity. Before 1930, the most actively sought result in graph theory was the characterization of planar graphs, and the characterization using K_5 and $K_{3,3}$ is known as Kuratowski's theorem. Kasimir Kuratowski's once asked Harary who originated the notation for K_5 and $K_{3,3}$; Harary replied that " K_5 stands for Kasimir and $K_{3,3}$ stands for Kuratowski!" Recall that *subdividing* an edge or performing an *elementary subdivision* means replacing the edge with a path of length 2. A subdivision of G is a graph obtained from G by a sequence of elementary subdivision, turning edges into paths through new vertices of degree 2. If $\delta(G)\geq 3$ and H is a subdivision of G, then vertices of H having degree at least 3 are the *branch vertices*; these are the images of the original vertices. Subdividing edges does not affect planarity, so we seek a characterization by finding the *topological minimal* non-planar graphs-those that are not subdivisions of other nonplanar graphs. We already know that a graph containing any subdivision of K_5 or $K_{3,3}$ is nonplanar. Kuratowski [1930] proved that G is planar if and only if G contains no subdivision of K_5 or $K_{3,3}$. Wagner [1937] proved another characterization. Deletion and contraction of edges preserve planarity, so we can seek the minimal nonplanar graphs using this operations. Wegner proved that G is planar if and only if it as no sub graph contractible to K_5 or $K_{3,3}$.

COLORING OF PLANAR GRAPHS: Because every simple planar graph with n vertices has at most $3n$–6 edges, every simple planar graph has a vertex of degree at most 5. Hence by induction planar graphs are 6 –colorable. Heawood improved this. The proof sounds like contradiction but is essentially induction.

Four color problem: The four-color theorem states that any map can be colored using four-colors in such a way that regions sharing a common boundary (other than a single point) do not share the same color. In 1976, Kenneth Appel and wolf-gang Haken established the four color theorem by using large scale computers. Earlier some proofs were given, which were contradicted later. The four color theorem states that every planar graph has a chromatic number four or less. For example, each of K_4 and wheel W_6 are four colorable. Because every simple planar graph with n vertices has at most $3n$-6. Edges, every simple planar graph has a vertex of degree at most 5. Hence by induction planar graphs are 6-colorable. Heawood improved this.

Heawood five color theorem, fourcolor problems and every planar graph is six colorable etc. are briefly discussed below.

THE FOUR-COLOR THEOREM AND THE HEAWOOD FIVE –COLOR THEOREM

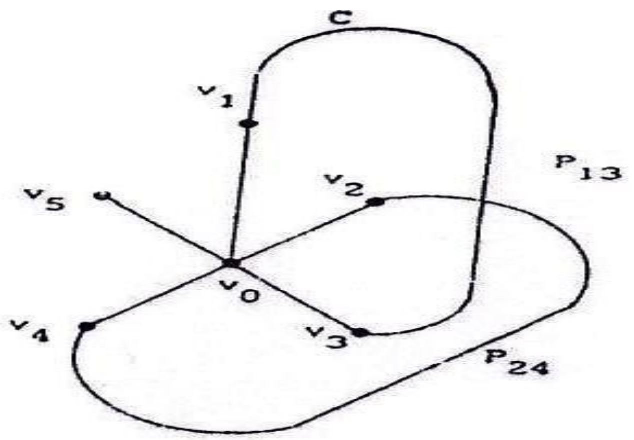
What is the minimum number of colors required to color the world map of countries so that no two countries having a common boundary receive the same color? This simple-looking problem manifested itself into one of the most challenging problems of graph theory. Popularly known as the four -color conjecture. The geographical map of the countries of the world is typical example f a plane graph. an assignment of colors to the faces of plan de graph g so the two faces having a common-boundary containing at least once edge receive the same color is a face coloring of G. The face chromatic number X'(G) of a plane graph G is the minimum k for which G has a face coloring using k-colors. The problem of coloring a map so that no two adjacent countries receive the same color can thus be transformed into a problem of face coloring of a plane graph G. The face coloring of G is closely related to the vertex coloring of G′ the fact that any two faces of g are adjacent in g if, the corresponding vertices of G′ are adjacent in G′ shows that G is k-face colorable if, and only if, G′ is k-vertex colorable. It was young Francis Guthrie who conjectured, while coloring the district map of England, that four colors were sufficient to color the world map so that adjacent countries receive distinct colors. This conjecture was communicated by his brother t de Morgan 1852. The conjecture of Guthrie is equivalent to the conjecture that any plane graph is 4-face colorable. The latter conjecture is equivalent to the conjecture every planar graph is 4-vertex colorable. Ever since the conjecture was first published in 1852, many eminent mathematicians especially graph theorists, attempted to settle the conjecture. In the process of settling the conjecture, many equivalent formulations of the conjecture were found. Assaults on the conjecture were made using such varied using such varied branches of mathematicians as algebra, number theory, and finite geometrics. the solution found the light of the day when K. Appel, W. Haken and J. Koch [4] of the university of established the validity of the conjecture in 1976 with the aid of computers see also reference [2] and [3]. The proof includes, among other things, 10^{10}units of operations, amounting to a staggering 1200 hours of computer time on the high-speed computer available at that time. Although the computer-oriented proof of Appel, Haken, and Koch settled the conjecture in 1976+ and has stood the rest of time, a theoretical proof of the fourcolor problem is still to be found. Even though the solution of the 4cc has been a formidable task, it is rather easy to establish that every planar graph is 6-vertex colorable,

Theorem: Every planar graph is 6-vertex colorable.

Proof: The proof is by induction on n, the number of vertices of the graph. The result is trivial for planar graphs with at most 6 vertices. Assume the result for planar graphs with n-1 n≥7 vertices. Let G be a planar graph with *n* vertices. By corollary $\delta(G) \leq 5$, and thus G has a vertex ν of degree at most 5. By hypothesis, G-ν is a 6-vertex colorable. In any 6-vertex coloring of G-ν the neighbors of ν in G would have used only at most five colors, and hence ν can be colored by an unused color. In other words, G is 6-vertex colorable. It involves some ingenious arguments to reduce the upper bound for the chromatic number of planar graph from 6 to 5. The upper bound 5 was obtained by Heawood [67] as early as 1890.

Theorem: (Heawood five color theorem) Every planar graph is 5-vertex colorable.

Proof: Proof is by induction on n(G) = *n* without loss of generality, we assume that G is a connected plane graph. If $n \leq 5$, the result is clearly true. Hence, assume that $n \geq 6$ and that any planar graph with fewer than *n* vertices is 5-vertex colorable. G being planar, $\delta(G) \leq 5$ and so G contains a vertex ν_o of degree not exceeding 5. By induction hypothesis. G-ν_o is 5-vertex colorable. If d $(\nu_o) \leq 4$, at most four colors would have been used in coloring the neighbors of ν_o in G in a 5-vertex coloring o G- ν_o hence, used color can then be assigned to ν_o to yield a proper 5-vertex coloring of G. If $d(\nu_o) = 5$, but only four or fewer colors are used to color the neighbors of ν_o in a proper 5-vertex coloring of G-ν_o, then also an unused color can be assigned to ν_0 to yield a proper 5-vertex coloring of G. Hence, Assume that the degree of ν_o in G and that in every 5-coloring of G- ν_o the neighbors of ν_o in cyclic order in a plane embedding of G. choose some proper 5-coloring of G- ν_o with colors. say $c_1, c_2, \ldots\ldots\ldots c_5$ Let $(v_1, v_2, \ldots\ldots v_5,)$ be the color partition of G- ν_o where the vertices in v_1 are colored c_i, $1 \leq i \leq 5$ assume further that $v_1 \in v_i$, $1 \leq i \leq 5$. Let G_{ij} be the sub graph of G- ν_o induced by $v_1 \cup v_j$ .Suppose v_1 and v_j, $1 \leq i, j \leq 5$, belong to distinct components of G_{ij} containing v_j would give a recoloring of G- ν_o in which only four colors are assigned to the neighbors of ν_o. But this against our assumption, Hence, $v_i$and v_j must belong to the same component of G_{ij}. Let P_{ij} be a v_i –-v_j path in G_{ij}. Let C denote the cycle $\nu_o \nu_1 P_{13} \nu_3 \nu_o$ in G (see the dia 4.1) . Then C separates ν_2 and ν_4 ; that is, one of ν_2 and ν_4 must lie in int C and the other in ext C. In the diagram $\nu_2 \in$ext C. Then P_{24} must cross C at a vertex of C. But this is clearly impossible since no vertex of C receives either of the colors c_2 and c_4. Hence, this possibility cannot arise and, G is 5-vertex colorable.



Note that the bound 4 in the inequality $\Psi(G) \leq 4$ for planar graphs G is best possible since K_4 is planar and $\Psi(K_4) = 4$.

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2. SOME APPLICATIONS OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT: Ordinary Differential Equations (ODE'S) are encountered in several areas of engineering, applied science, finance and biology etc. In this paper we present and discuss the some applications of first order ordinary differential equations like particle moving on a curve and water tanks.

INTRODUCTION: Ordinary Differential Equations (ODE'S) are encountered in several areas of engineering, applied science, finance and biology etc.

Definition: A differential equation is an equation which relates some unknown function, say, to one or more of its derivatives.

Definition: A first order differential equation involves only the first derivative of the function. A higher order differential equation involves higher derivatives. For instance, a differential equation relating y to its second derivative (and perhaps its first derivative as well) would be a second order differential equation.

$y\frac{dy}{dx} = x$ and $y - 3\frac{dy}{dx} + 4 = 0$ are first order differential equations

$y + \frac{dy}{dx} - \frac{d^2y}{dx^2} = 3x^2 - 2$ is a second order differential equation.

We will sometimes use the abbreviation D.E. for the phrase differential equation

WATER TANKS: Suppose we have a container full of a salt solution of a certain concentration. If we pump in a solution with a different concentration at a certain rate, mix well and extract the mixture at the same rate, how does the concentration change with time? To solve this, we consider the law of conservation of salt.

$$\begin{pmatrix}\text{Rate of change of salt}\\ \text{in the container}\end{pmatrix} = \begin{pmatrix}\text{Rate of inflow of salt}\\ \text{into the container}\end{pmatrix} - \begin{pmatrix}\text{Rate of outflow of salt}\\ \text{from the container}\end{pmatrix}$$

Example 1: A tank contains a salt water solution consisting initially of 20 kg of salt dissolved into 10 *l* of water. Fresh water is being poured into the tank at a rate of 3 *l*/min and the solution (kept uniform by stirring) is flowing out at 2 *l* /min. Figure 3.4 shows this setup. Find the amount of salt in the tank after 5 minutes.

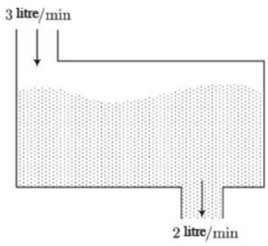


Figure : Fresh water is being poured into the tank as the well-mixed solution

Solution: Let Q(t) be the amount (in kilograms) of salt in the tank at time t (in minutes). The volume of water at time t is

Q $10 + t$

$$10 + 3t - 2t = 10 + t.$$

The concentration at time t is given by

$$\frac{\text{amount of salt}}{volume} = \frac{Q}{10 + t}$$ kg per litre. Then

$$\frac{dQ}{dt} = -(\text{rate at which salt is leaving}) = -\frac{Q}{10+t}.2 = -\frac{2Q}{10+t}$$

Thus, the solution to the problem is the solution of

$$\frac{dQ}{dt} = -\frac{2Q}{10 + t}$$

evaluated at t = 5. We see that it is simply a separable equation.

To solve it, we have

where $A = e^C$. But $Q \geq 0$ (we cannot initially, i.e., at t = 0, we know

that the tank contains 20 kg of salt. Thus, the initial condition is

Q (0) = 20, and we have A = 2000. −2

Figure shows a plot of the solution. Evaluating at t = 5 gives

$Q(5) = 2000 / 15^2 = 8.89$.

Therefore, after 5 minutes, the tank will contain approximately

8.89 kg of salt.

have a negative amount of salt) and $t \geq 0$(we do not visit the past), so we remove absolute value signs, giving us $Q = A / (10 + t)^{-2}$.

 ISBN: 978-93-5437-830-0

Example 2: A 1000 ltr tank of water initially contains 10 kg of dissolved salt. A pipe brings a salt solution (concentration 0.005 kgl^{-1}) into the tank at a rate of 2 ls^{-1}, and a second pipe carries away the excess solution. Calculate C (t), the concentration of salt, assuming that the tank is well mixed.

Solution: Let V = 1000 ltr be the volume of water in the tank (this is constant) and let m(t) be the mass of salt dissolved in the water, so m(0) = 10 kg.

Let C(t) = m/V be the concentration of salt in the water, assuming that the salt is well mixed, So C(0) = 0.01 kgl^{-1}. The rate of inflow of salt (in kg/s) is $C_{in}r_{in}$, where C_{in} = 0.005 kgl^{-1} is the concentration of salt in the inflow, and r in = 2 ls^{-1} is the rate of in flow. The rate of outflow of salt (in kg/s) is $C_{out}r_{out}$, where C_{out} = C(t) is the concentration of salt in the outflow, equal to the concentration of salt in the water in the tank in this case, and $r_{out} = r_{in}$ = 2 ls^{-1} is the rate of outflow, equal to the rate of inflow in this case. From the law of concentration of salt, we get

$$\frac{dm}{dt} = C_{in}\, r_{in} - C_{out}\, r_{out}$$

Now make use of the fact that m(t)

$$V\frac{dC}{dt} = (C_{in} - C)r_{in} \quad \text{or} \quad \frac{dC}{dt} + \frac{r_{in}}{V}C = \frac{r_{in}}{V}C_{in}$$

This is a first - order linear ODE; its integrating factor is exp ($-r_{in}$/v) t and the solution in this case is

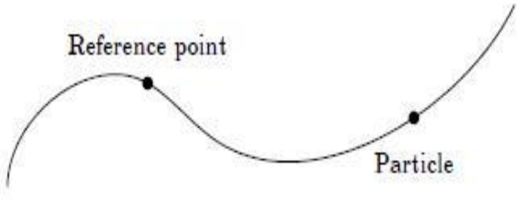
$$C(t) = C_{in} + K\exp\left(\frac{-r_{in}}{V}t\right),$$

Where k is constant. Putting in the initial condition and the numbers results in:

C (t) = 0:005 + 0:005$e^{-0.002t}$

Where C is in kgl^{-1} and t is in seconds.

Particle Moving on a Curve: Let x(t) and y(t) denote the x and y coordinates of a point at time t, respectively. Pick a reference point on the curve and let s(t) be the arc length of the



piece of the curve between the reference point and the particle. Figure illustrates this idea. Then

Example : A particle is placed at the point (1, 1) on the curve $y = x^3$ and released. It slides down the curve under the influence of gravity. Determine whether or not it will ever fly off the curve, and if so, where.

Solution: First note that

$y = x^3$, $y^1 = 3x^2$, $y^{11} = 6x$, $y_0 = 1$.

Therefore,

$2y^{11}(y - y_0) = 1 + (y^1)^2$,

$3x^4 - 12x - 1 = 0$.

Solving for the relevant value of x gives us x = −0.083, which corresponds to y = −0.00057. Therefore, the particle will fly off the curve at (−0.083,−0.00057).

Figure shows the graph

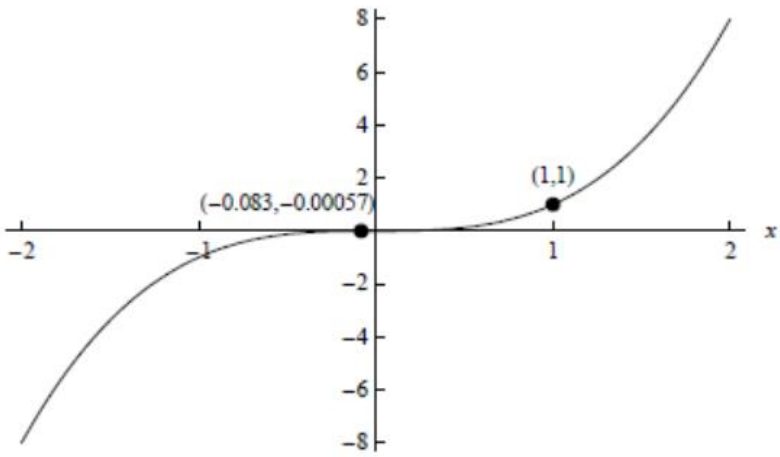


Figure : A particle released from (1, 1), under the influence of gravity, will fly off the curve at approximately (−0.083,−0.00057).

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3. APPLICATION OF GRAPH IN NEURAL NETWORKS

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Abstract: Many underlying relationships among data in several areas of science and engineering, e.g., computer vision, molecular chemistry, molecular biology, pattern recognition, and data mining, can be represented in terms of graphs. In this paper, various types of neural networks are explained and demonstrated, applications of neural networks like ANNs in medicine are described, and a detailed historical background is provided. The connection between the artificial and the real thing is also investigated and explained. Finally, the mathematical models involved are presented and demonstrated.

Keywords: Underlying relationships, molecular chemistry, pattern recognition, computer vision, artificial neural networks

Introduction: Data can be naturally represented by graph structures in several application areas, including image analysis [1], scene description [2], [3], software engineering [4], [5], and natural language processing [6]. The simplest kinds of graph structures include single nodes and sequences. But in several applications, the information is organized in more complex graph structures such as trees, acyclic graphs, or cyclic graphs. Traditionally, data relationships exploitation has been the subject of many studies in the community of inductive logic programming and, recently, this research theme has been evolving in different directions [7], also because of the applications of relevant concepts in statistics and neural networks to such areas as in [8]–[11].

An Artificial Neural Network (ANN) is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. The key element of this paradigm is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. ANNs, like people, learn by example. An ANN is configured for a specific application, such as pattern recognition or data classification, through a learning process. Learning in biological systems involves adjustments to the synaptic connections that exist between the neurons. Neural networks can be used to extract patterns and detect trends that are too complex to be noticed by either humans or other computer techniques. A trained neural network can be thought of as an "expert" in the category of information it has been given to analyse. This expert can then be used to provide projections given new situations of interest and answer "what if" questions. Other advantages include:

1. Adaptive learning: An ability to learn how to do tasks based on the data given for training or initial experience.
2. Self-Organization: An ANN can create its own organization or representation of the information it receives during learning time.
3. Real Time Operation: ANN computations may be carried out in parallel, and special hardware devices are being designed and manufactured which take advantage of this capability.
4. In the human brain, a typical neuron collects signals from others through a host of fine structures called dendrites. The neuron sends out spikes of electrical activity through a long, thin stand known as an axon, which splits into thousands of branches. At the end of each branch, a structure called a synapse converts the activity from the axon into electrical effects that inhibit or excite activity from the axon into electrical effects that inhibit or excite activity in the connected neurons. When a neuron receives excitatory input that is sufficiently large compared with its inhibitory input, it sends a spike of electrical activity down its axon. Learning occurs by changing the effectiveness of the synapses so that the influence of one neuron on another changes. It is shown in figure 1

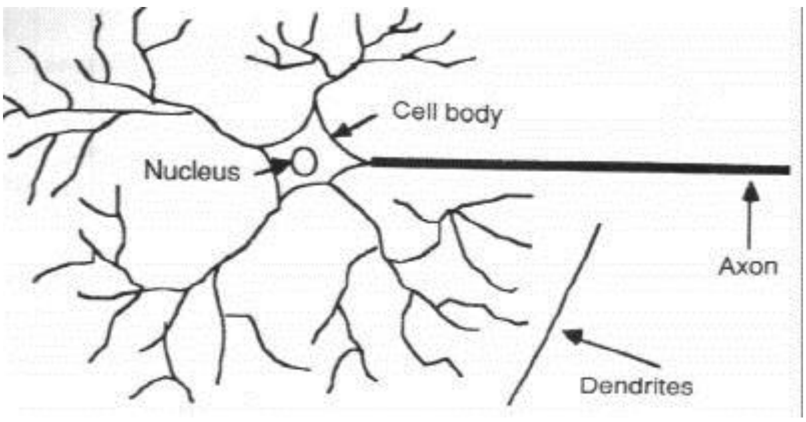


Fig 1 Componenets of neurons

Artificially neurons are synthesized the why human neurons works and is implemented. An artificial neuron is a device with many inputs and one output. The neuron has two modes of operation; the training mode and the using mode. In the training mode, the neuron can be trained to fire (or not), for particular input patterns. In the using mode, when a taught input pattern is detected at the input, its associated output becomes the current output. If the input pattern does not belong in the taught list of input patterns, the firing rule is used to determine whether to fire or not. A simple neuron is shown in the figure 2.

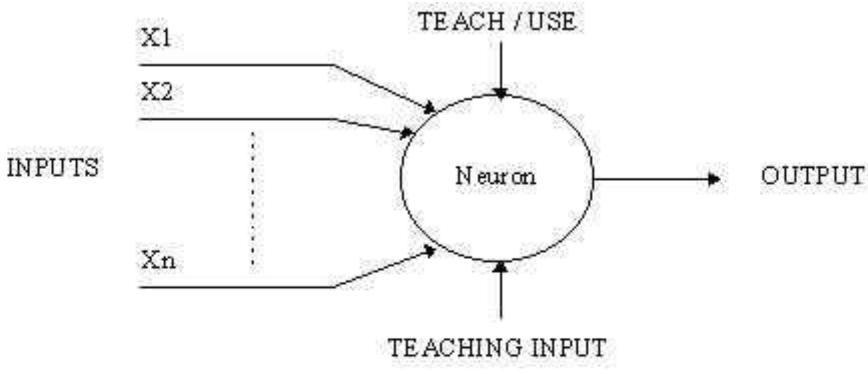
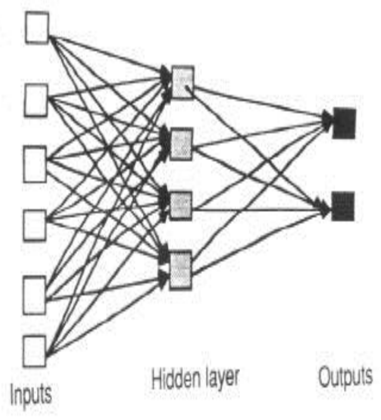


Fig 2 A simpole neuro

Feed-forward Artificial Neural Networks are little forward networks (fig.3) that associate inputs with outputs. They are extensively used in pattern recognition. This type of organization is also referred to as bottom-up or top-down. Whereas, Feedback networks (fig4) can have signals travelling in both directions by introducing loops in the network. Feedback networks are very powerful and can get extremely complicated. Feedback networks are dynamic; their 'state' is changing continuously until they reach an equilibrium point. They remain at the equilibrium point until the input changes and a new equilibrium needs to be found. Feedback architectures are also referred to as interactive or recurrent, although the latter term is often used to denote feedback connections in single-layer organizations



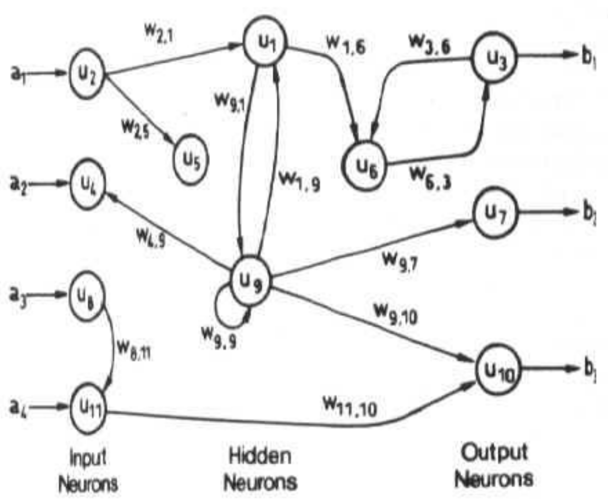


Figure 3 Feed forword network Figure 4 : Feedback networks

The most influential work on neural nets is' perceptrons' a term coined by Frank Rosenblatt. The perceptron (fig 5) turns out to be an MCP model (neuron with weighted inputs) with some additional, fixed, pre-processing. Units labelled A1, A2, Aj, Ap are called association units and their task is to extract specific, localised featured from the input images.

Perceptrons mimic the basic idea behind the mammalian visual system. They were mainly used in pattern recognition even though their capabilities extended a lot more.

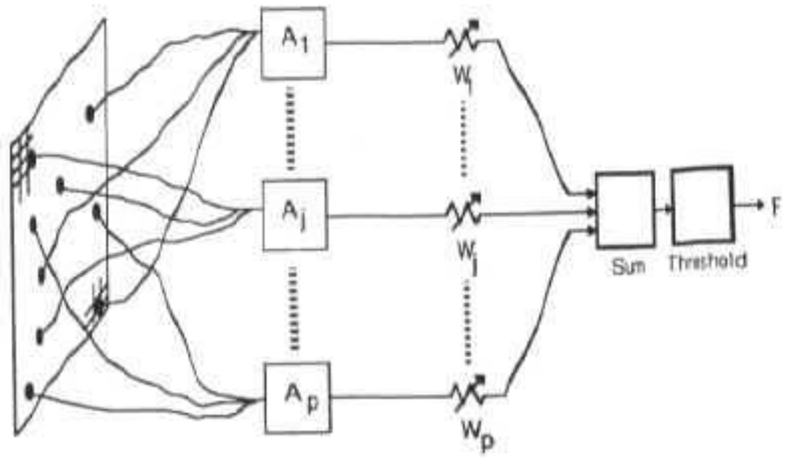


Fig 5 Perceptrons.

The memorization of patterns and the subsequent response of the network can be categorized into two general paradigms:

Associative mapping in which the network learns to produce a particular pattern on the set of input units whenever another particular pattern is applied on the set of input units. The associative mapping can generally be broken down into two mechanisms:
Auto-association: an input pattern is associated with itself and the states of input and output units coincide. This is used to provide pattern completion, i.e., to produce a pattern whenever a portion of it or a distorted pattern is presented. In the second case, the network actually stores pairs of patterns building an association between two sets of patterns.
Hetero-association: is related to two recall mechanisms:

Nearest-neighbor recall, where the output pattern produced corresponds to the input pattern stored, which is closest to the pattern presented, and

Interpolative recall, where the output pattern is a similarity dependent interpolation of the patterns stored corresponding to the pattern presented. Yet another paradigm, which is a variant associative mapping, is classification, i.e. when there is a fixed set of categories into which the input patterns are to be classified.

Regularity detection in which all units learn to respond to a particular properties of the input patterns. Whereas in associative mapping the network stores the relationships among patterns, in regularity detection the response of each unit has a particular 'meaning'. This type of learning mechanism is essential for feature discovery and knowledge representation.

Every neural network possesses knowledge which is contained in the values of the connections weights. Modifying the knowledge stored in the network as a function of experience implies a learning rule for changing the values of the weights.

Neural Networks in Practice

Neural networks have broad applicability to real world business problems. In fact, they have already been successfully applied in many industries. Since neural networks are best at identifying patterns or trends in data, they are well suited for prediction or forecasting needs including: 1. Sales forecasting. 2. Industrial process control 3. Customer research 4. Data validation 5. Risk management 6. Target marketing

ANN are also used in the following specific paradigms: recognition of speakers in communications; diagnosis of hepatitis; recovery of telecommunications from faulty software; undersea mine detection; texture analysis; three-dimensional object recognition; hand-written word recognition; and facial recognition.

Neural networks in medicine

Artificial Neural Networks (ANN) are currently a 'hot' research area in medicine and it is believed that they will receive extensive application to biomedical systems in the next few years. At the moment, the research is mostly on modelling parts of the human body and recognizing diseases from various scans (e.g. cardiograms, CAT scans, ultrasonic scans, etc.).

Neural networks are ideal in recognizing diseases using scans since there is no need to provide a specific algorithm on how to identify the disease. Neural networks learn by example so the details of how to recognize the disease are not needed. What is needed is a set of examples that are representative of all the variations of the disease. The quantity of examples is not as important as the 'quantity'. The examples need to be selected very carefully if the system is to perform reliably and efficiently.

Modelling and Diagnosing the Cardiovascular System

Neural Networks are used experimentally to model the human cardiovascular system. Diagnosis can be achieved by building a model of the cardiovascular system of an individual and comparing it with the real time physiological measurements taken from the patient. If this routine is carried out regularly, potential harmful medical conditions can be detected at an early stage and thus make the process of combating the disease much easier.

A model of an individual's cardiovascular system must mimic the relationship among physiological variables (i.e., heart rate, systolic and diastolic blood pressures, and breathing rate) at different physical activity levels. If a model is adapted to an individual, then it becomes a model of the physical condition of that individual. The simulator will have to be able to adapt to the features of any individual without the supervision of an expert. This calls for a neural network.

Another reason that justifies the use of ANN technology is the ability of ANNs to provide sensor fusion which is the combining of values from several different sensors. Sensor fusion enables the ANNs to learn complex relationships among the individual sensor values, which would otherwise be lost if the values were individually analysed. In medical modelling and diagnosis, this implies that even though each sensor in a set may be sensitive only to a specific physiological variable, ANNs are capable of detecting complex medical conditions by fusing the data from the individual biomedical sensors.

Electronic noses

ANNs are used experimentally to implement electronic noses. Electronic noses have several potential applications in telemedicine. Telemedicine is the practice of medicine over long distances via a communication link. The electronic nose would identify odours in the remote surgical environment. These identified odours would then be electronically transmitted to another site where in door generation system would recreate them. Because the sense of smell can be an important sense to the surgeon, telesmell would enhance telepresent surgery.

Instant Physician

An application developed in the mid-1980s called the "instant physician" trained an auto associative memory neural network to store a large number of medical records, each of which includes information on symptoms, diagnosis, and treatment for a particular case. After training, the net can be presented with input consisting of a set of symptoms; it will then find the full stored pattern that represents the "best" diagnosis and treatment.

Neural Networks in business

Business is a diverted field with several general areas of specialisation such as accounting or financial analysis. Almost any neural network application would fit into one business area or financial analysis.

There is some potential for using neural networks for business purposes, including resource allocation and scheduling. There is also a strong potential for using neural networks for database mining that is, searching for patterns implicit within the explicitly stored information in databases. Most of the funded work in this area is classified as proprietary. Thus, it is not possible to report on the full extent of the work going on. Most work is applying neural networks, such as the Hopfield-Tank network for optimization and scheduling.

Marketing

There is a marketing application which has been integrated with a neural network system. The Airline Marketing Tactician (a trademark abbreviated as AMT) is a computer system made of various intelligent technologies including expert systems. A feed forward neural network is integrated with the AMT and was trained using back-propagation to assist the marketing control of airline seat allocations. The adaptive neural approach was amenable to rule expression. Additionally, the application's environment changed rapidly and constantly, which required a continuously adaptive solution. The system is used to monitor and recommend booking advice for each departure. Such information has a direct impact on the profitability of an airline and can provide a technological advantage for users of the system. [Hutchison & Stephens, 1987]

While it is significant that neural networks have been applied to this problem, it is also important to see that this intelligent technology can be integrated with expert systems and other approaches to make a functional system. Neural networks were used to discover the influence of undefined interactions by the various variables. While these interactions were not defined, they were used by the neural system to develop useful conclusions. It is also noteworthy to see that neural networks can influence the bottom line.

Credit Evaluation

The HNC Company, founded by Robert Hecht-Nielsen, has developed several neural network applications. One of them is the Credit Scoring systems which increase the profitability of the existing model up to 27%. The HNC neural systems were also applied to mortgage screening. Neural network automated mortgage insurance underwriting system was developed by the Nestor Company. This system was trained with 5048 applications of which 2597 were certified. The data related to property and borrower qualifications. In a conservative mode the system agreed on the under writers on 97% of the cases. In the liberal model the system agreed 84% of the cases. This is system run on an Apollo DN3000 and used 250K memory while processing a case file in approximately 1 sec.

Conclusion

The computing world has a lot to gain from neural networks. Their ability to learn by example makes them very flexible and powerful. Furthermore there is no need to devise an algorithm in order to perform a specific task; i.e. there is no need to understand the internal mechanisms of that task. They are also very well suited for real time systems because of their fast response and computational times which are due to their parallel architecture. Neural networks also contribute to other areas of research such as neurology and psychology. They are regularly used to model parts of living organisms and to investigate the internal mechanisms of the brain. Perhaps the most exciting aspect of neural networks is the possibility that someday 'conscious' networks might be produced. There is a number of scientists arguing that consciousness is a 'mechanical' property and that 'conscious' neural networks are a realistic possibility. Finally, I would like to state that even though neural networks have a huge potential we will only get the best of them when they are integrated with computing, AI, fuzzy logic and related subjects

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4. MAXIMA CODE TO CHECK THE SOLUTION OF SECOND ORDER ODES WHEN THE COEFFICIENTS ARE FUNCTIONS OF INDEPENDENT VARIABLE

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ABSTRACT

The laws of the natural and physical world are usually written and modeled in the form of differential equations. Most of them are second order differential equations. Many methods of solving differential equations are available. An attempt is made here to write a code for the solution of the second order differential equation with variable coefficients using maxima. Four methods considered are when a part of the complementary function is given, changing the dependent variable, variation of parameters and solution by checking the exactness. The code serves as a quick method to verify the correctness of the solutions obtained when the differential equations are solved manually.

Keywords: Second order ODE, Variable coefficients, Maxima.

1. INTRODUCTION

A differential equation is an equation involving derivatives of an unknown function and possibly the function itself as well as the independent variables. Differential equations have a remarkable ability to predict the world around us. They are used in a wide variety of disciplines from biology, economics, physics, chemistry and engineering. They can describe exponential growth and decay, the population growth of species or the change in investment return over time and many other physical situations.

Most ordinary differential equations have no known exact solution (or the exact solution is a complicated expression involving many terms with special functions) and one normally uses approximate methods. However some ordinary differential equations (ODEs) have simple exact solutions, and many of these can be found using *ode2*, *desolve,* or *contrib ode* in the MAXIMA open source software. Maxima is a computer algebra system based on a 1982 version of Macsyma. It is written in Common Lisp and runs on all POSIX platforms such as macOS, Unix, BSD, and Linux, as well as under Microsoft Windows and Android. It is free software released under the terms of the GNU General Public License. Maxima is a system for the manipulation of symbolic and numerical expressions including differentiation, integration, Taylor-series, Laplace transforms, ODEs, system of linear equations, polynomials, sets, lists, vectors, matrices and tensors. Maxima, yields high precision numerical results by using exact fractions, arbitrary-precision integers and variable-precision floating-point numbers. It can plot functions and data in two or three dimensions. The Maxima source code can be compiled on many systems including Windows, Linux and MacOS X. The source code for all systems and precompiled binaries for Windows and Linux are available at the SourceForge file manager.

The calling sequence to get the solution of an ODE is *ode2 (eqn, dvar, ivar).*The function *ode2*solves an ordinary differential equation (ODE) of first or second order. It takes three Arguments: an ODE given by *eqn*, the dependent variable *dvar*, and the independent variable *ivar*. When successful, it returns either an explicit or implicit solution for the dependent variable. *% c*is used to represent the integration constant in the case of first-order equations, and*%k1* and *%k2* the constants for second-order equations. The dependence of the dependent variable on the independent variable does not have to be written explicitly, as in the case of *desolve*, but the independent variable must always be given as the third argument.

In this paper an attempt has been made to solve the differential equation using the maxima code with necessary explanation. Four methods of solving second order differential equations with variable coefficients are considered in the following four sections.

2. Solution of second order ODE with variable coefficients.

The general form of a second order ODE is of the form $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$ (1)

Where *P*, *Q, R*are functions of *x*. Equation (1) is solved for the following four different cases.

2.1. When a part of a complementary function is given.

Working rule:

Step 1: If R=0 then the equation (1) is homogeneous and the solution would be $y = c_1\phi_1$(x) +$c_2\phi_2$(x).

Step 2: If one of the two functions ϕ_1(x) and ϕ_2(x) is known, let it be *u(x)*. Complete solution of equation (1) can be sought as *y=uv*, where *v=v(x)* is to be determined.

Step 3: Now $y' = uv' + vu'$, $y'' = uv'' + 2u'v' + vu''$ substituting these in equation (1) and using the fact that *u(x)* is a part of the complementary function, the equation reduces to $uv'' + (2u' + Pu)v' = R$, by putting $w=\frac{dv}{dx}$ we get

$$u\frac{dw}{dx} + \left(2\frac{du}{dx} + Pu\right)w = R \text{ or } \frac{dw}{dx} + p(x)w = q(x) \qquad (2)$$

Where $p(x) = \frac{2}{u}\frac{du}{dx} + P$ and $q(x) = \frac{R}{u}$.Equation (2) is of first order and can be solved for *w* which on integration yields *v(x).* Thus *y=uv* gives the solution of equation (1)

Maxima program to solve $x^2y'' + xy - 9 = 0$ when $y = x^3$ is a part of the complementary function

```
kill(all)$
P:1/x$
Q:-9/x^2$
R:0$
u:x^3$
eqn:'diff(v,x,2)+(P+2/u*diff(u,x))*'diff(v,x)=R/u$
soln1:ode2 (eqn,v,x);
Soln:y=fullratsimp(u*rhs(soln1));
```

OUTPUT

(soln1) $v = \frac{\%k2}{x^6} + \frac{\%k1}{6}$

(Soln) $y = \frac{\%k1\, x^6 + 6\, \%k2}{6\, x^3}$

In the above program *kill (all)* is used to erase all the previously defined variables (in case they were defined). The *$*sign will suppress the output. Comparing the given ODE with the standard equation (1), the values of *P, Q, R* and *u* (part of the complementary function) are defined in the second , third , fourth and fifth line of the program respectively. The *eqn* in the sixth line is the equation (2) as obtained in the step 3 of the working rule. *Ode2* is used to solve linear differential equations in the seventh line and the solution is assigned to the variable *soln1*. Lastly the solution obtained in the previous line is simplified using the command *fullratsimp* and is assigned to the variable name *Soln.* The solution thus obtained in the last step matches with the solution obtained by solving the given ODE using the steps described in the working rule above.

2.2. By changing the dependent variable

Working rule:

Step 1: The standard form is as given by equation (1).Complete solution of equation (1) can be sought as $y = uv$, where $u = u(x)$ and $v = v(x)$ have to be chosen appropriately. Substituting $y'$and y" in equation (1), we get

$$uv'' + (2u' + Pu)v' + (u'' + Pu' + Qu)v = R \quad (3)$$

Step 2: Choose $u = u(x)$ such that $2u' + Pu = 0$ (4)

Because of which first derivative term in equation (3) vanishes. From equation (4) we get $u = e^{-\frac{1}{2}\int P dx}$ using this and the expressions u' and u" which can be obtained from equation (4), equation (3) reduces to $\frac{d^2v}{dx^2} + Iv = \frac{R}{u}$ (5)

Step 3: Where $I = Q - \frac{1}{4}P^2 - \frac{1}{2}P'$

Where equation (5) is called normal form which can be solved when I is a constant or of the form $\frac{K}{x^2}$where K is a constant.

Maxima program to solve $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right)y = 0$ **by reducing to normal form**

```
kill(all)$
P:-2/x$
Q:1+2/x^2$
R:0$
u:%e^(-1/2*(integrate(P,x)));
I:Q-1/4*P^2-1/2*diff(P,x)$
eqn:'diff(v,x,2)+I*v=R/u$
soln1:fullratsimp(ode2(eqn,v,x));
Soln:y=fullratsimp(u*rhs(soln1));
```

OUTPUT

```
(u)      x
(soln1)  v= %k1 sin(x) + %k2 cos(x)
(Soln)   y= %k1 x sin(x) + %k2 x cos(x)
```

The maxima code for this method is as per the steps explained in the working rule. First four lines of the program are similar as explained in section 2.1. As per the step 2 of this section u is defined in fifth line. Sixth line of the program is defined as per step 3 of this section. THe remaining lines in the program are similar to the program in section 2.1. Here the solution of the ODE is fully simplified and then assigned to the variable *soln1*.

2.3. By the method of variation of parameters

Working rule:

Step 1: Given equation (1), find complementary function in the form

$$y_c = c_1 y_1 + c_2 y_2 \tag{6}$$

Step 2: Assume the complete solution of equation (1) in the form $y = v_1 y_1 + v_2 y_2$ by changing c_1 and c_2 to v_1 and v_2 in equation (6).

Step 3: Write down the equations $v_1' y_1 + v_2' y_2 = 0$ and $v_1' y_1 + v_2' y_2 = R$ and solve for v_1'

and v_2', we get $v_1' = \frac{-y_2 R}{w}$; $v_2' = \frac{y_1 R}{w}$ where *w* is the Wronskian of y_1 and y_2 defined by the determinant $w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$.

Step 4: On integrating v_1' and v_2' obtained in step 3 to get v_1 and v_2. Put these values into $y = v_1 y_1 + v_2 y_2$, we get the complete solution of equation (1).

Maxima program to solve $(D^2 + a^2)y = \sec ax$ by the method of variation of parameters

```
kill(all)$
R:sec(a*x)$
assume(a>0)$
eqn:'diff(y,x,2)+a^2*y=0$
CF:ode2(eqn,y,x);
y1:coeff(part(rhs(CF),2),%k2)$
y2:coeff(part(rhs(CF),1),%k1)$
w:trigsimp(y1*diff(y2,x)-y2*diff(y1,x));
v1:trigsimp(integrate(-y2*R/w,x))+c1;
v2:trigsimp(integrate(y1*R/w,x))+c2;
Soln:subst([%k2=v1,%k1=v2],rhs(CF));
```

OUTPUT

(CF) $y = \%k1 \sin(a\,x) + \%k2 \cos(a\,x)$

(w) a

(v1) $\frac{\log(\cos(a\,x))}{a^2} + c1$

(v2) $\frac{x}{a} + c2$

(Soln) $\cos(a\,x)\left(\frac{\log(\cos(a\,x))}{a^2} + c1\right) + \left(\frac{x}{a} + c2\right)\sin(a\,x)$

The code here is as per the working rule. The function on the RHS of the given ODE is assigned to the variable *R* as in the previous sections. For simplicity the value of the parameter *a* is assumed to be positive. The fourth line of the program is the LHS of the given equation. Fifth line gives the complementary function (*CF* here) as can be seen in the output. The sixth and seventh line of the program are the coefficients of the constants *%k2*and *%k1* (as *cosax*is taken as y_1 and *sinax* as y_2). The corresponding codes simply imply that the coefficient *%k2* in second term of RHS of *CF* is assigned to y_1 and similarly y_2. The eighth line is the expansion of the Wronskian as explained in step 3 of this section. Here *trigsimp* command is used as the expansion involves trigonometric functions**.** The ninth and tenth lines give the values of $v_1$and v_2 as described in the step 4. The last line of the program is to obtain the solution by substituting these values of v_1 and v_2. The command *subst* will do this task.

2.4. When the equation is exact

Working rule:

Consider the equation $a_0y''+a_1y'+a_2y=f(x)$ (7)

where a_0, a_1 and a_2 are functions of *x*.

$F(D,D')z=0$, where $F(D,D')=a_0D^2+a_1DD'+a_2D'^2$

Step 1: Check for the exactness of equation (1) using $a_0''-a_1'+a_2=0.$

Step 2: Then the first integral is $a_0y'+\left(a_1-a_0'\right)y=\int f(x)\,dx+c.$

Step 3: The solution of the equation obtained in step2 is the complete solution of equation (1)

Maxima program to solve $x^2y''+3xy'+y=\dfrac{1}{(1-x)^2}$

```
kill(all)$
A0:x^2$
A1:3*x$
A2:1$
f:1/(1-x)^2$
flag:0$
if (diff(A0,x,2)-diff(A1,x)+A2)#0 then
flag:1$
if flag=1 then
disp("Equation is not exact")
else
disp("Equation is exact")$
if flag=0 then
eqn:'diff(y,x)+(A1-diff(A0,x))/A0*y    =(integrate(f,x)+c)/A0;
Soln:ratsimp(ode2(eqn,y,x));
```

OUTPUT

Equation is exact

(Soln) $y=\dfrac{(c+1)\log(x)-\log(x-1)+\%c}{x}$

The second, third, fourth and fifth lines of the program are the values of a_0, a_1, a_2 and $f(x)$. A new variable *flag* is used as the conditional statement is true then it gives the equation as not exact which is assigned to *flag=1*. Hence when *flag=0*, the equation will be exact and the solution can be obtained as in the earlier sections.

3. CONCLUSION

The maxima code serves as a quick method to verify the correctness of the solutions obtained when the differential equations solved manually.

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5. ON PULL BACK VECTOR BUNDLES

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Abstract: In this paper, given a vector bundle ξ over a topological space X and a continuous map f: Y → X, we construct a pullback vector bundle f (ξ) over. We prove that given a vector bundle ξ over X and homotopic continuous maps f_0, f_1:Y → X, the induced pull back bundles f_0 (ξ) and f_1 (ξ) are isomorphic if Y is compact Hausdroff.

1. Introduction:

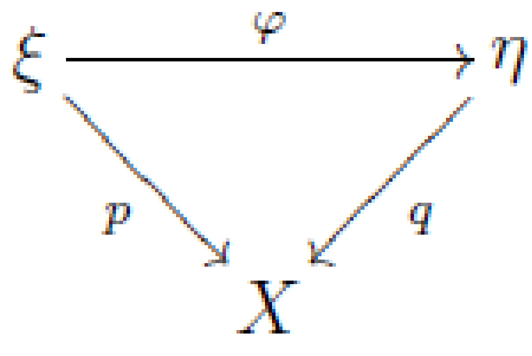
Definition 1.1. Let X be a topological space. A vector bundle ξ over X is a triplet (ξ, p, X) together with

1. a continuous onto map $p : \xi \to X$
2. a finite dimensional vector space structure on each

$$\xi_x = p^{-1}(x) \text{ forx} \in X$$

The map p is called the projection map, the space ξ is called total space of the family, the space X is called the base space of the family, and if $x \in X$, ξ_x is called the fibre over x.

Definition 1.2. A homomorphism from one family $p : \xi \to X$ to another familyq $: \eta \to X$ is a continuous map $\phi : \xi \to \eta$ such that:



Example 1.7. *Let ξ and η be vector bundles over X. Then $Hom(\xi, \eta) = \bigcup_{x \in X} Hom(\xi_x, \eta_x)$ is a vector bundle, where $Hom(\xi_x, \eta_x)$ is the vector space of all linear maps from ξ_x to η_x.*

Remark 1.8. *The bundle $Hom(\xi, \xi)$ is denoted by $End(\xi)$.*

2 Main Result

Theorem 2.1. *Given a continuous map $f : Y \to X$ and vector bundle $p : \xi \to X$, there exists a vector bundle $p' : \xi' \to Y$ with a map taking the fiber of ξ' over each*

point y in Y isomorphically onto the fiber of ξ over $f(y)$, and such vector bundle ξ' is unique up to isomorphism.

Proof. Let $\xi = (\xi, p, X)$ be a vector bundle and let $f : Y \to X$ be a continuous map from a space Y to X. Consider the subspace $\xi' = \{(y, e) \in Y \times \xi \mid f(y) = p(e)\}$ of the product space $Y \times \xi$. With respect to the map $p' : \xi' \to Y$ given by $p'(y, e) = y$, $\xi' = (\xi', p', Y)$ is a vector bundle. For,
Let us define $g : \xi' \to \xi$ by $g((y, e)) = e$. It is easy to see that g is a homomorphism. Moreover, g maps the fiber of ξ' over each point y in Y isomorphically onto the fiber of ξ over $f(y)$.

$$\begin{array}{ccc} \xi & \longrightarrow & X \\ {\scriptstyle (g\circ f)'}\uparrow & & \uparrow{\scriptstyle g\circ f} \\ (g \circ f)^*(\xi) & \longrightarrow & Z \end{array}$$

Thus τ is an isomorphism and so ξ' is isomorphic to ξ''.
We often denote ξ' as $f^*(\xi)$ and we call it a **pullback bundle** of ξ. If ξ is a vector bundle then the local triviality of $f^*(\xi)$ follows from the local triviality of ξ. To show this, let us consider the local triviality $\phi : p^{-1}(U) \to U \times V$, where U is a neighborhood of $f(y), y \in Y$. Then $f^{-1}(U)$ is a neighborhood of y. We define

$$\psi : p'^{-1}(f^{-1}(U)) \to f^{-1}(U) \times V$$

by

$$\psi(y_1, e) = (y_1, \phi_2(e)),$$

where ϕ_2 is given by ϕ and $\phi(e) = (\phi_1(e), \phi_2(e)) = (p(e), \phi_2(e))$. Clearly ψ is a homeomorphism and for each $y_1 \in f^{-1}(U)$, $\psi_{y_1} : p'^{-1}(y_1) \to \{y_1\} \times V$ is a vector space isomorphism. □

Lemma 2.2. *Given a pair of continuous maps $Z \xrightarrow{f} Y \xrightarrow{g} X$ and a vector bundle $\xi =$*

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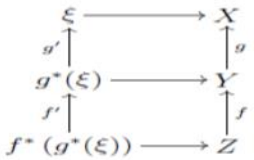
Lemma 2.2. *Given a pair of continuous maps $Z \xrightarrow{f} Y \xrightarrow{g} X$ and a vector bundle $\xi =$*

and

$$\begin{aligned} f^*(g^*(\xi)) &= \{(z, v) \in Z \times g^*(\xi) \mid f(z) = q(v)\} \\ &= \{(z, (y, v)) \in Z \times (Y \times \xi) \mid g(y) = p(v), f(z) = y\} \\ &= \{z, (f(x), v) \mid g(f(z)) = p(v)\}. \end{aligned}$$



The map $(z, u) \to (z, (f(z), u))$ is a vector bundle isomorphism from $(g \circ f)^*(\xi)$ to $f^*(g^*(\xi))$. □

Lemma 2.3 (A bundle form of the Tietze extension theorem). *Let X be a compact Hausdorff space, $Y \subset X$ a closed subspace, and ξ a vector bundle over X. Then any section $s : Y \to \xi|_Y$ can be extended to X.*

Proof. Proof follows by [1, Lemma 1.4.1]. □

Lemma 2.4. *Let X be a compact Hausdorff space, $Y \subset X$ a closed subspace, and let ξ, ζ be two vector bundles over X.If $f : \xi|_Y \to \zeta|_Y$ is an isomorphism, then there exist an open set U containing Y and an extension $t_f : \xi|_U \to \zeta|_U$ of f which is an isomorphism.*

Proof. Since $f : \xi|_Y \to \zeta|_Y$ is an isomorphism, f is a section of $\mathrm{Hom}(\xi|_Y, \zeta|_Y)$, and thus extends to a section t_f of $\mathrm{Hom}(\xi, \zeta)$. Let U be the set of those points for which this map t_f is a vector bundle isomorphism. Then U is open and contains Y. This proves the theorem. □

Lemma 2.5. *Let Y be a compact Hausdorff space, $f_t : Y \to X$ $(0 \leq t \leq 1)$ a homotopy and let ξ be a vector bundle over X. Then $f_0^*(\xi) \cong f_1^*(\xi)$.*

Proof. Let I denotes the unit interval $[0, 1]$, $f : Y \times I \to X$ be the homotopy, so that $f(y, t) = f_t(y)$ and let $\pi : Y \times I \to Y$ denote the projection. Now we apply the Lemma (2.4) to the vector bundles $f^*(\xi)$, $\pi^* f_t^*(\xi)$ and the subspace $Y \times \{t\}$ of $Y \times I$, on which there is an obvious isomorphism φ. By the compactness of Y, we deduce that $f^*(\xi)$ and $\pi^* f_t^*(\xi)$ are isomorphic in some strip $Y \times U_t$ where U_t denotes a neighbourhood of $\{t\}$ in I. Hence the isomorphism class of $f_t^*(\xi)$ is a locally constant function of t. Since I is connected this implies it is constant, hence $f_0^*(\xi) \cong f_1^*(\xi)$. □

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6. FOURIER SERIES AND ITS APPLICATIONS

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ABSTRACT: The Fourier Series, the founding principle behind the field of Fourier Analysis, is an infinite expansion of a function in terms of sines and cosines. In physics and engineering, expanding functions in terms of sines and cosines is useful because it allows one to more easily manipulate functions that are, for example, simply difficult to represent analytically. In particular, the fields of electronics, quantum mechanics, and electrodynamics all make heavy use of the Fourier series. Additionally, other methods based on the Fourier Series, such as the FFT (Fast Fourier Transform – a form of a Discrete Fourier Transform [DFT]), are particularly useful for the fields of Digital Signal Processing (DSP) and Spectral Analysis.

Introduction:

By Njegos Nincic

- Transforms
    - Mathematical Introduction
- Fourier Transform
    - Time-Space Domain and Frequency Domain
    - Discrete Fourier Transform
- Fast Fourier Transform: Applications
- Summary

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt$$

- References

Transforms: In mathematics, a function that results when a given function is multiplied by a so-called kernel function, and the product is integrated between suitable limits. (Britannica)

$$F(s) = \{\mathcal{L}f\}(s) = \int_{0^-}^{\infty} e^{-st} f(t)\, dt.$$

- Can be thought of as a substitution

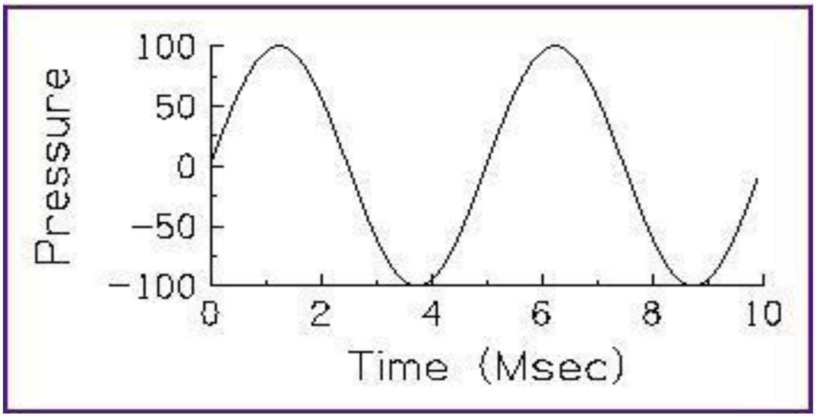
- Example of a substitution:
- Original equation: $x + 4x^2 - 8 = 0$
- Familiar form: $ax^2 + bx + c = 0$
- Let: $y = x^2$
- Solve for y
- $x = \pm\sqrt{y}$

Property of transforms: They convert a function from one domain to another with no loss of information. Fourier Transform converts a function from the time (or spatial) domain to the frequency domain.

Time Domain and Frequency Domain

- Time Domain:

  Tells us how properties (air pressure in a sound function, for example) change over time:

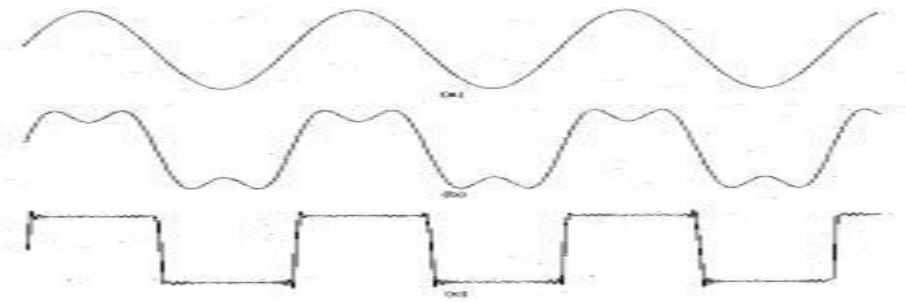


Amplitude = 100

Frequency = number of cycles in one second = 200 Hz

Time Domain and Frequency Domain

- In 1807, Jean Baptiste Joseph Fourier showed that any periodic signal could be represented by a series of sinusoidal functions
- In picture: the composition of the first two functions gives the bottom one



Example:

Human ears do not hear wave-like oscillations, but constant t one

Often it is easier to work in the frequency domain.

Fourier Transform

- Because of the property

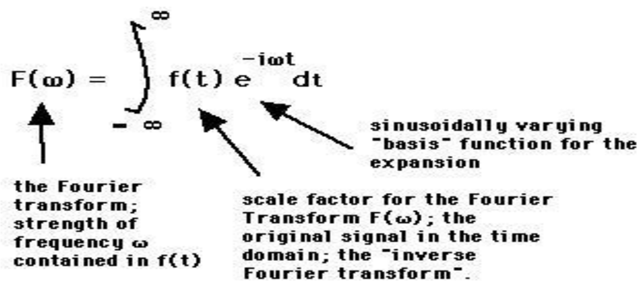
EULER's FORMULA

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$\text{where } i = \sqrt{-1}$$

Transform takes us to the frequency domain:



 ISBN: 978-93-5437-830-0

Discrete Fourier Transform

$$f_j = \sum_{k=0}^{n-1} x_k e^{-\frac{2\pi i}{n} jk} \qquad j = 0, \ldots, n-1$$

- O(n²) time complexity

Fast Fourier Transform

- Many techniques introduced that reduce computing time to O(n log n)
- Most popular one: **radix-2** decimation-in-time (**DIT**) FFT Cooley-Tukey algorithm:

$$\begin{aligned} f_j &= \sum_{k=0}^{\frac{n}{2}-1} x_{2k} e^{-\frac{2\pi i}{n} j(2k)} + \sum_{k=0}^{\frac{n}{2}-1} x_{2k+1} e^{-\frac{2\pi i}{n} j(2k+1)} \\ &= \sum_{k=0}^{n'-1} x'_k e^{-\frac{2\pi i}{n'} jk} + e^{-\frac{2\pi i}{n} j} \sum_{k=0}^{n'-1} x''_k e^{-\frac{2\pi i}{n'} jk} \\ &= \begin{cases} f'_j + e^{-\frac{2\pi i}{n} j} f''_j & \text{if } j < n' \\ f'_{j-n'} - e^{-\frac{2\pi i}{n}(j-n')} f''_{j-n'} & \text{if } j \geq n' \end{cases} \end{aligned}$$

Applications

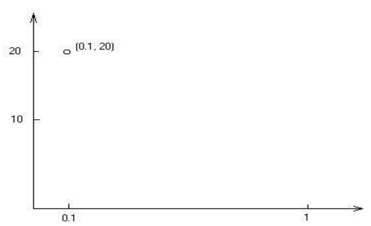
- In image processing:

  Instead of time domain: *spatial domain* (normal image space)

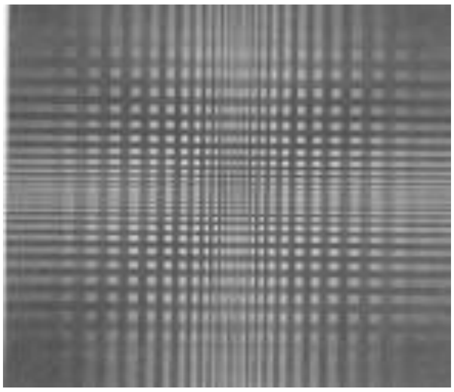
  Frequency domain: space in which each image value at image position F represents the amount that the intensity values in image I vary over a specific distance related to F

Applications: Frequency Domain In Images

- If there is value 20 at the point that represents the frequency 0.1 (or 1 period every 10 pixels). This means that in the corresponding spatial domain image I the intensity values vary from dark to light and back to dark over a distance of 10 pixels, and that the contrast between the lightest and darkest is 40 gray levels



Spatial frequency of an image refers to the rate at which the pixel intensities change



In picture on right:

High frequencies: Near centre

Low frequencies: Corners

Applications: Image Filtering Summary:

- Transforms:

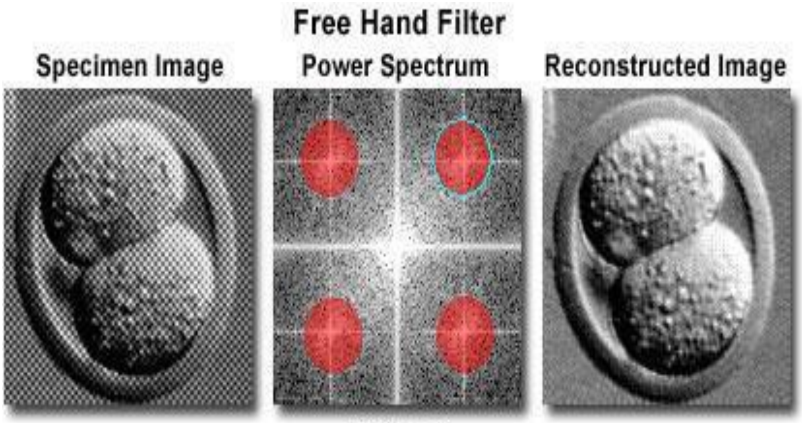


Figure 1

Useful in mathematics (solving DE)

- Fourier Transform: Lets us easily switch between time-space domain and frequency domain so applicable in many other areas. Easy to pick out frequencies any applications

Advanced noise cancellation and cell phone network technology uses Fourier series where digital filtering is used to minimize noise and bandwidth demands respectively

Fourier Transform infra-red and FT-Raman spectroscopy, nuclear magnetic resonance (a basic tool in chemistry but more familiar in medicine via MRI imaging), and x-ray diffraction from crystals, the ultimate tool for determining molecular structure.

1. Signal Processing. It may be the best application of Fourier analysis.
2. Approximation Theory. We use Fourier series to write a function as a trigonometric polynomial.
3. Control Theory. The Fourier series of functions in the differential equation often gives some prediction about the behaviour of the solution of differential equation. They are useful to find out the dynamics of the solution.
4. Partial Differential equation. We use it to solve higher order partial differential equations by the method of separation of variables.
5. Hiss and pop in sound recordings can be cleaned up using Fourier analysis. What is static but a super-high frequency sound--higher than most sounds that normally appear in music and speech, etc. When a time-domain signal is represented in the frequency domain, i.e., as a sum of sine waves, you can cure the static by simply erasing all the highest frequencies and then reconstituting the sound.
6. Exactly the same trick works in removing speckles from photographs. The boundaries between the photo and the speckles are the highest frequency components of the image. Using Fourier analysis to you can drop all the highest frequency components. Then reconstitute the picture, and like magic, speckles are gone. Some minor features you might want will be gone too, but in practice it works very well.
7. The opposite works for finding outlines in a picture. Erase everything but the high frequencies, and when you reconstitute the picture, you will have all the outlines and the broad areas will all be erased.
8. The Fourier series of functions in the differential equation often gives some prediction about the behaviour of the solution of differential equation. They are useful to find out the dynamics of the solution.
9. The original motivation for Fourier series was to approximate any periodic function, such as square waves and triangle waves, by sines and cosines. So they naturally come up in acoustics, where the multitude of sounds you hear through your headphones are built up from these sinusoidal functions, which are themselves absurdly simple to generate.

10. Yet Fourier series are not perfect and their approximation of a square wave is an example. Even after including a lot of sines and cosines, there still remains an overshoot/undershoot at the transition between high and low amplitudes:
11. Fourier series underpin the answers to these questions. While the
12. Fourier **transform** represents a procedure to convert between the continuous-wave and discrete samples, the **series** itself remains relevant to the analysis of how well this transformation goes both ways.
13. Other applications of the actual Fourier series in physics and engineering include the general heat equation (which was the problem Fourier himself solved with the series that got named after him), vibrational modes of structural elements in buildings, quantum harmonic oscillators and generally any place where some function repeats itself over and over.

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7. INFLUENCE OF GRAPH THEORY IN ALLIED FIELDS

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ABSTRACT: Graphs are discrete structures consisting of vertices and edges that connect these vertices. In the real world, there are many problems that can be represented with the help of a graph. Here, we have explored the importance of graphs in various fields. In this paper, an overview is presented to demonstrate its importance in multiple fields which includes Pure Mathematics, Operation Research, Computer Science, Bio Chemistry, Sociology and other fields.

Keywords: Graph, Vertices, Edges, Operations Research, Bio Chemistry etc.

INTRODUCTION

A network and its connectivity can be represented symbolically in terms of a graph. But for practical propose, it is concerned that how networks can be encoded to get the desired results when implemented and how to ease out for proper utility by end users. A graph is an ordered pair of set of vertices V and set of edges E. Vertices can be presented with the points in a plane and these data types are called nodes. A line connecting these nodes is called an edge. If a graph is directed then edges are ordered pairs and if a graph is undirected then its edges are unordered pairs. The order of the graph is equal to the number of vertices in it. Leonhard Euler became the father of graph theory when he settled a famous unsolved problem called the Konigsberg Bridge Problem. Graphs has many features like data flow diagram, decision making ability, displays relationships among objects, easy alterations and modifications in existing system etc.

AREAS OF GRAPH THEORY

A concept of graph theory is widely growing and moving into the mainstream of mathematics as it is playing a vital role in following areas:

- In Pure Mathematics
- In Computer Science
- In Operation Research
- In Sociology
- In Science

IMPORTANCE IN PURE MATHEMATICS

Cantor Schroder-Bernstein Theorem

Graph theory is used in this theorem. This theorem states that for two sets A and B, if there is an injective mapping from A to B and an injective mapping from B to A then there is a one-one and onto mapping from A to B i.e. A and B have the same number of elements. Now, A and B are two disjoint sets. Each component is either a one-way infinite path or a two way infinite path. There is a set of edges in each component such that each vertex is incident with one of these edges. In this, bipartite graph is used. Therefore, in each component, the vertices from A have same number of elements as the subset of vertices from B.

INFLUENCE IN COMPUTER SCIENCE

Data Mining: Graph theory plays a important role in data mining as graph mining. Graph mining describes the relational aspect of the data. The different approaches of graph mining are Sub graph categories, Sub graph isomorphism, Mining measures, Solution methods and invariants.

Data Structures: The structuring or organizing of data into information so that operations like traversing, searching, sorting, merging, insertion, deletion, etc. becomes easy, Such logical and mathematical model is called as 'Data Structures' [5]. The choice of data structuring model depends upon two factors:

- It must depict actual relationship among data.
- The structure should be simple and easy to process the data into information if required. The nonlinear representation of data into memory is possible using graph theory.

Arbitrary relationship among data is represented by a graph and its adjacency matrix. Many graph algorithms requires traversing the nodes and edges of a graph systematically. There are two standard ways to traverse a graph:

- Breadth – First search
- Depth – First Search

The Breadth – First search technique uses queue data structure and Depth – First search technique uses stack data structure.

Software Engineering: Graphs are widely used in engineering the software at every level of the engineering model. The various engineering models are waterfall model, Spiral model, Prototype model, Iterative model etc. Following is the role of graph theory at levels of software engineering models:

Phases	Role of Graph Theory
Requirement Analysis and Specification	Data Flow Diagrams (DFD)
Design Phase	Graphical design is used for depicting relationship among modules
Testing	Control flow of a program associated with McCabe's complexity which describes directed graphs for representing the sequence of instructions executed.

Operating System: An operating system[6] is a program that acts as an interface between user and computer hardware. The purpose of operating system is to provide an environment in which a user can execute programs in an efficient and convenient manner. Graph theory plays an important role in operating system in solving job scheduling and resource allocation problems. The concept of graph coloring is applied in job scheduling problems of CPU. Jobs are assumed as vertices of a graph and there will be an edge between two jobs that cannot be executed simultaneously. Graphs are also used in disk scheduling algorithms.

Website Designing: The graph theory is used to model the website designing process, where web pages are represented by vertices and the hyper links between them are represented by edges in the graph. This concept is known as web graph. In graph theory such a graph is called as complete bipartite graph. Graph representation helps in finding all connected components and using directed graph we can evaluate website utility and link structure.

IMPORTANCE IN OPERATION RESEARCH

Time Table Management Problem: Graphs are widely used in solving Timetable management problem. Let us assume that in a school we have N teachers and M subjects, suppose we need a particular teacher to teach a particular subject in the respective period and authority needs to prepare a time table using the minimum possible number of periods. This is Timetable management problem. It can be solved by using Bipartite multiple graphs. To solve this problem we partitioned vertex set into two disjoint subsets. One is for teachers and other is for subjects. The relationship among both subsets is represented by edges.

Road Maps: Graphs can be used to demonstrate road maps. In road maps, Intersecting points are represented by vertices and roads are represented by edges. One way roads are represented by directed graphs and two way roads are represented by undirected graphs. Multiple undirected edges represent multiple two way roads connecting the same two intersections. The multiple one way roads that start at one intersection and end at other intersection are represented by multiple directed edges. The road having same starting and terminal point are represented by loops. Therefore to represent road maps, mixed graphs are required.

Travelling Sales Man Problem: Graphs can be used in travelling sales man problem to visit a route. For these we use Hamiltonian paths and circuits. Assuming a salesman has to visit *'n'* cities. He wants to start from a particular city and visit each city once and returning to his home city without visiting single city twice. His main purpose is to select the sequence in which the cities are visited in such a manner that his total travelling time or distance is minimized.

PERT AND CPM: Graph theory is also used widely in scheduling the project tasks. The most popular and successful applications of graphs in operation research is the planning and scheduling of large complicated projects. The best well known problems are PERT (Project Evaluation and Review Technique) and CPM (Critical Path Method)[8]. CPM and PERT are operation research techniques that were developed in the late 1950. Once the activity network representation has been worked out, resources are allocated to each activity. Resource allocation is typicallydone using a Gantt chart. After resource allocation is done, a Project Evaluation and Review Technique (PERT) chart representation is developed. A path in the activity network graph is any set of consecutive nodes and edges in this graph from the starting node to the last node. A critical path consists of set of dependent tasks that need to be performed in a sequence and which together take the longest time to complete.

IMPORTANCE IN SOCIOLOGY

Acquaintanceship Graph: Graphs are used to represent various relationships between people. For Example Simple graph is used to represent the acquaintance relationship among each other that whether they are known to each other or not. Each person is represented by a vertex and relationship is represented by edges. An undirected graph depicts a relationship that they are known to each other.

Influence Graph: It is observed that some people can influence the thinking of others; in this case directed graphs are used. Each person can be represented as a vertex and there is a directed edge when the person influences the other person. It does not contain loops and multiple edges. For Example, in a college Principal can influence teachers and teachers can influence students.

Measuring performance and Progress report: Graph theory is widely used in an organization to measure the performance of an employee for certain period. The graph can depict the progress or degrading of an employee or region. It will help an employee to motivate and work with more dedication.

Other Applications: Possible applications include targeted advertising, identifying leaders of terrorist networks, central hubs in transportation networks, or dominant species in an ecosystem.

IMPORTANCE IN SCIENCE

Niche overlap graph in Ecology: Graphs are used to represent the interaction of different species of animals. In an ecosystem, the competition between the species can be explained by niche overlap graph. Each species is represented as a vertex and an undirected edge represents that species compete. It does not contain loops and multiple edges.

Graphs in Chemistry: Graphs are used in the field of chemistry to model chemical compounds, study of molecules, construction of bonds and study of atoms. In computational biochemistry some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modelled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices, (sequences) to eliminate all conflicts. In brief, graph theory has its unique impact in various fields and is growing large now a days. The subsequent section analyses the applications of graph theory especially in computer science.

Bioinformatics – DNA fragment assembly: The Swiss biochemist Frederich Miescher first observed DNA in the late 1869. DNA is De oxyribo Nucleic Acid is a molecule that is found in every living organism. It contains the instruction an organism needs to develop, live and reproduce. These instructions are found in every cell, and are assed down from parents to their children.DNA[9] is made up of molecules called nucleotides. Each nucleotide contains a phosphate group, a sugar group and a nitrogen base. These four types of nitrogen bases are Adenine (A), Thymine (T), Guanine (G) and Cytosine (C). DNA instructions are determined on the order of these bases. DNA sequencing and fragment assembly is the problem of reconstructing full strands of DNA based on the pieces of data recorded. The Eulerian circuit in Graph theory is implemented to solve the problem of DNA fragment assembly.

In Medical Sciences: The Graphs are also widely used in medical ultrasound projections. Medical ultrasound images is an important type of medical images and is widely used in medical diagnosis, Compared with other medical imaging methods, ultrasound imaging has the advantages of non-traumatic to human body, real-time display, low cost, ease to use. Now a day's 3D imaging techniques are widely used in medical sciences. The graph theory is used to construct a 3D graph is called 3D graph based segmentation algorithm. It can generate a set of minimum spanning trees each of which corresponds to a 3D sub region. Graph based segmentation model is much far better than 3D active contour model (3D snake) as it is less complex therefore results are computed in less time. It is more accurate as well.

CONCLUSION

The objective of this paper is to bring in the notice of readers and research scholars about the importance and the relevance of the graph theory in every area like pure mathematics, Computer science, sociology, operation research and other science applications. There are many other interesting application areas where graph theory has played a vital role. This work is presented especially to project the applications of idea of graph theory in few areas so that the readers can take forward to implement in new areas

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8. A STUDY ON STATISTICAL TOOL USED IN SOCIAL SCIENCE RESEARCH IN TAMILNADU

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Abstract: Social Science research has been updating himself every day, the similar title of research has been provided a different result in a different circumstance, because of findings and conclusion of research have been derived from the people. The people's culture, time, behaviors, Nation, Environment, and all surrounding of respondents influence the result of social Research and very Social science influence the Production or Manufacturing, Distribution, logistics of the Products and Services. Social science research has the power of creating desires into needs and all major decision of corporate has been taken based on research. Hence Social Science needs the help of Statistics. Statistics help to understand the raw data of the research. Statistics also facilitate the tabulation of data, finding central tendency, dispersion, correlation, regression, hypothesis testing, time serious, index, etc.

INTRODUCTION

"Without numbers nothing in the world" Currency in ultimate for business, it measured on the base of Numerical number, in the business we cannot express very things in word and recordable, hence the number is important in Social science and easy way to display the growth of business and doing business. Every statistical tool has been expressing the data as simplified and understandable. Accounting's purpose is to get information about the business to make the decision, that information must be understandable by all, hence the statistical tools have been a help to us express all data us we like it and comparison data and making decisions in the organization. Good social science research must use relevant statistic tools to provide a good result of the research because all good social science research has been improved the lifestyle of people and making a profit to the company.

Statement of the problem: In India 865 universities are available, in South India 187 universities like Andhra Pradesh 34, Karnataka 60, Kerala 17, Puducherry 2, Tamilnadu 52, and Telangana 22. All universities, government, and Aid College have Research centres and doing Research. Hence every research needs the support of statistics. Here we will analyse how the statistical tools have been used in Social Science.

The emphasis of concepts: The present study concentrated has been carried out in south India, social science research plays a vital role in the market; today market depended on creating needs and selling the product like today's internet packing. Hence the result of very research has been important to industries to retain the customer and doing business for that statistical tool has been a help to us getting an accurate result of the research.

The objective of the study

The following are the objective of the current study.

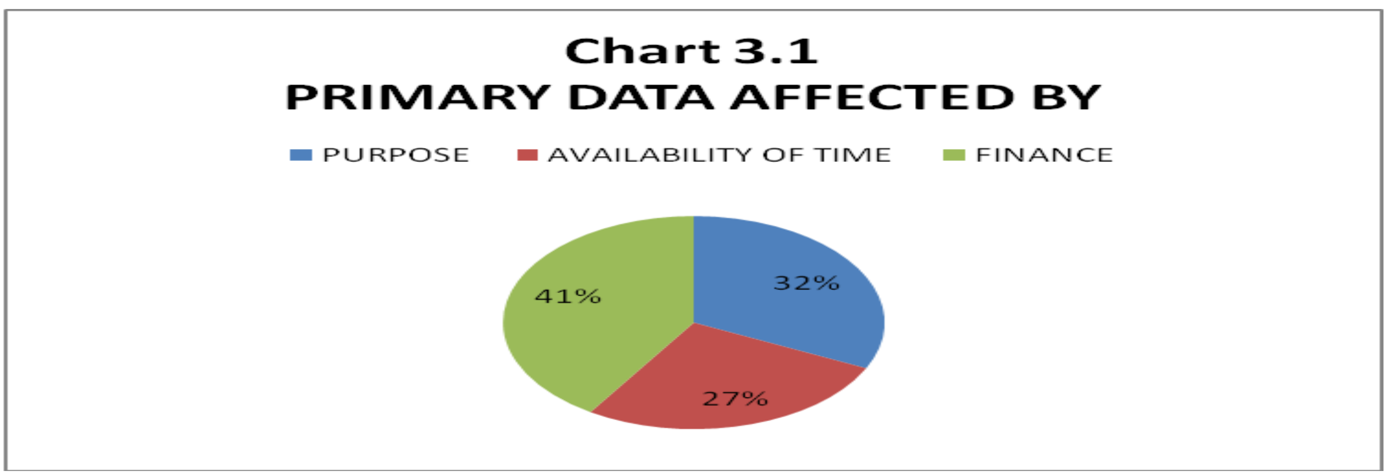
- To identify the frequently using Statistical Tool research.
- To examine the best Statistical Tool in Research.
- To find out factors affected the researchers to use Statistical Tool.

RESEARCH METHODOLOGY

As the population is unbounded hence adopting the convenient sampling method, the researcher has selected 300 respondents as the sample for the study area to collect information. The study is in mutual aid with primary and secondary data. To collect primary data thought Google forms. The questions in the Google form were pre-test and modified before finalization. The secondary data were collected from Websites, Journals, Books, and Magazines.

Analysis of data

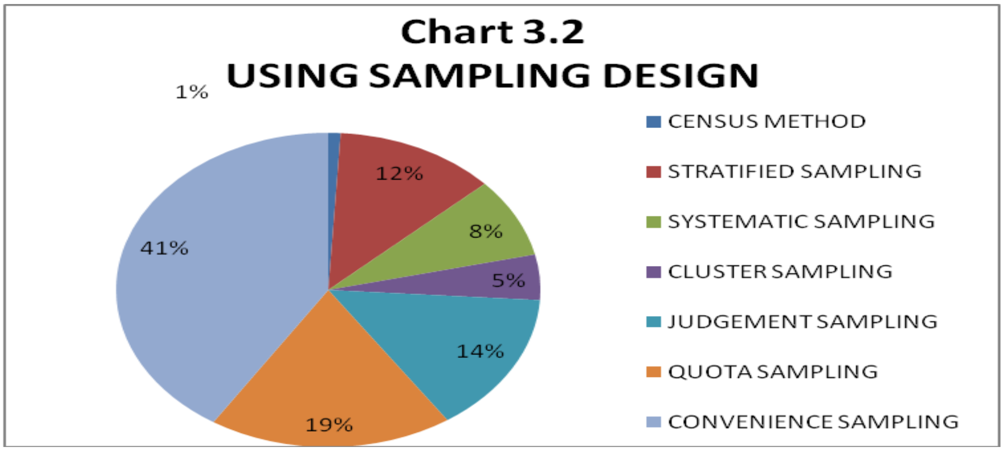
Collection of Data: Data collection is the important work of every research most commonly data will be collected through primary and secondary data. The collected data has been called raw data as per the convenience of the research use tabulation and simple percentage analysis methods convert the raw data to useful information.



Above chart has been clear that primary data has been affected by the purpose of the research it means the research finds some more important things at the time of investigation but cannot be added that because of time, finance, etc.

USING SAMPLING DESIGN

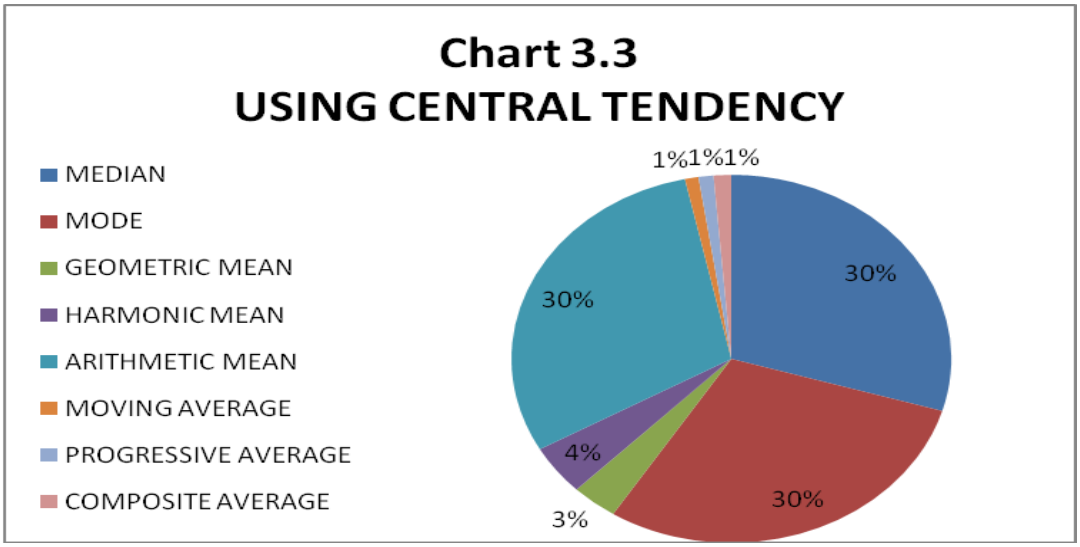
The investigator not known the extract population of the research area and unable collected the data from each person, hence the sampling help to doing research in defined small group and result will be applied to the entire population



When sampling size increase automatically error will be minimized, the above chart clearly showed that most of the research use convenience sampling its increase bias and increase the error of the research.

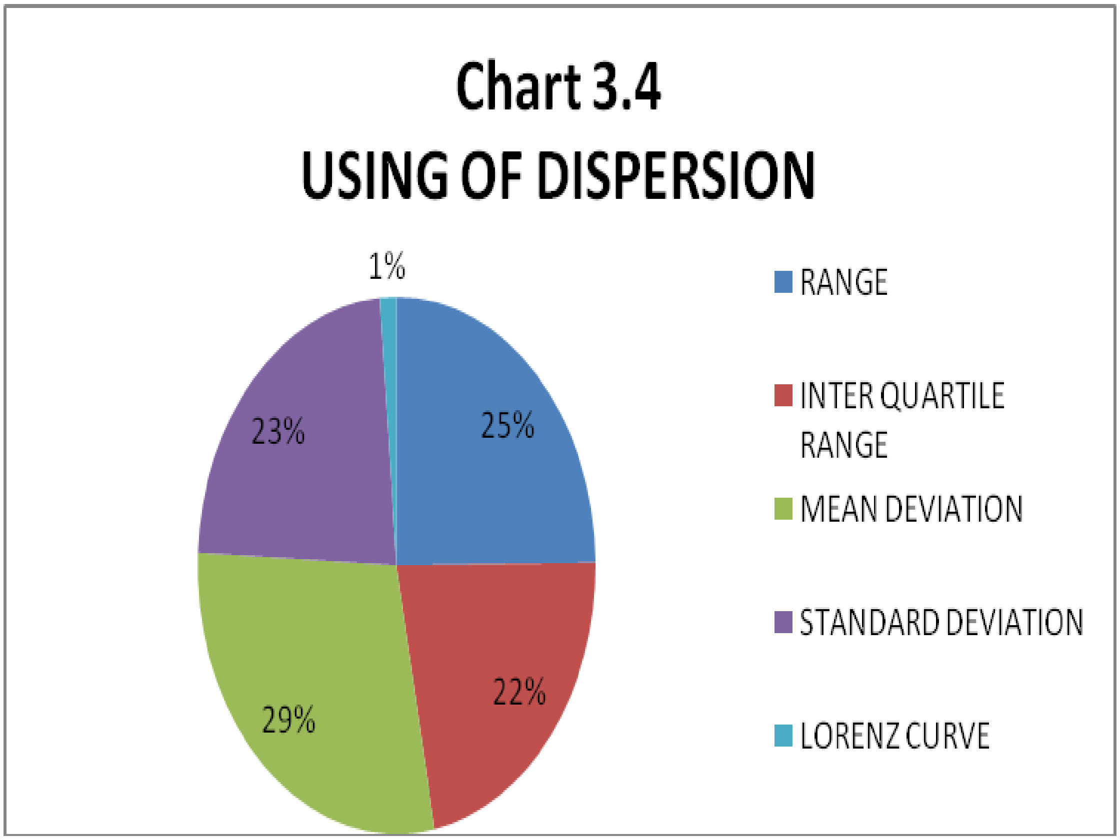
USING CENTRAL TENDENCY

Find central tendency is common for all research, as we ask 10^{th} 12^{th} percentage. Here most commonly 60% of the researchers have been used median and Arithmetic mean. Help to make average growth of industries.



USING OF DISPERSION

Reliability of central tendency (mean, mode, median). The variation was less than mean it good. If the variation is more than mean it means no use of the data.



The above chart indicates that 29% of respondents use mean deviation for analysis dispersion off the data.

USING SKEWNESS

Skewness is helping to know the relationship of mean, median, and mode. The skewness distribution may be positive or negative.

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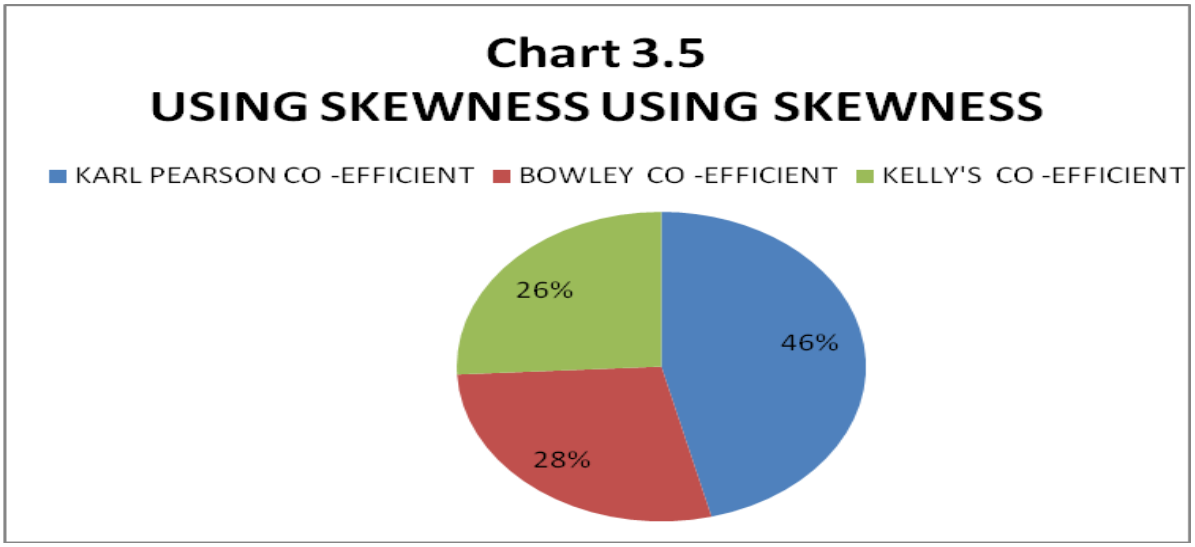
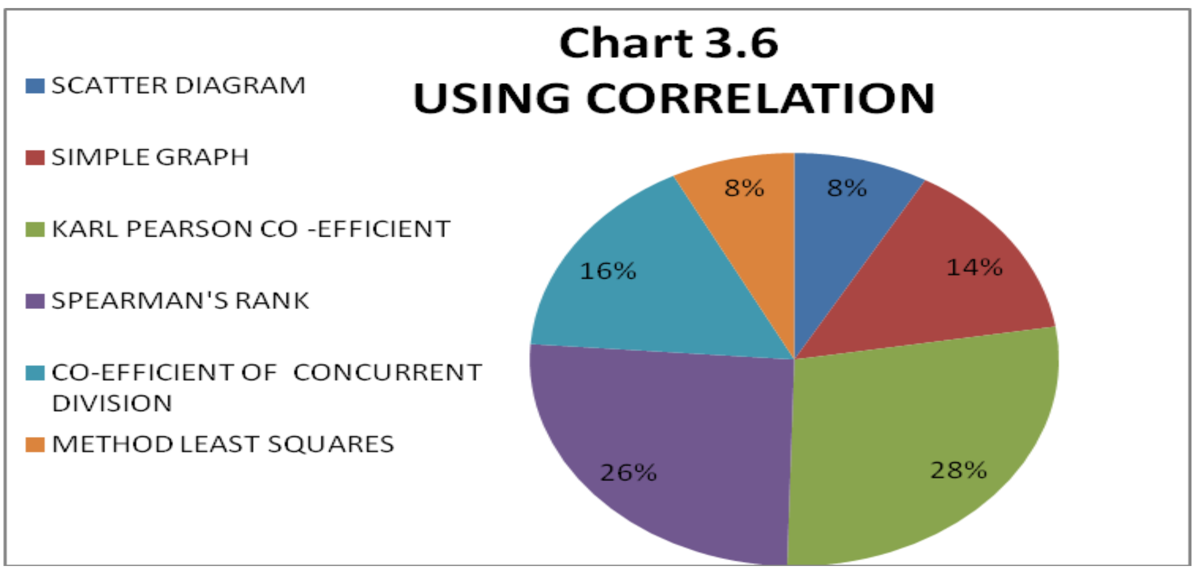
 ISBN: 978-93-5437-830-0



Though Chart 3.5 we could find 46% of respondents are using Karl Pearson co-efficient.

USING CORRELATION

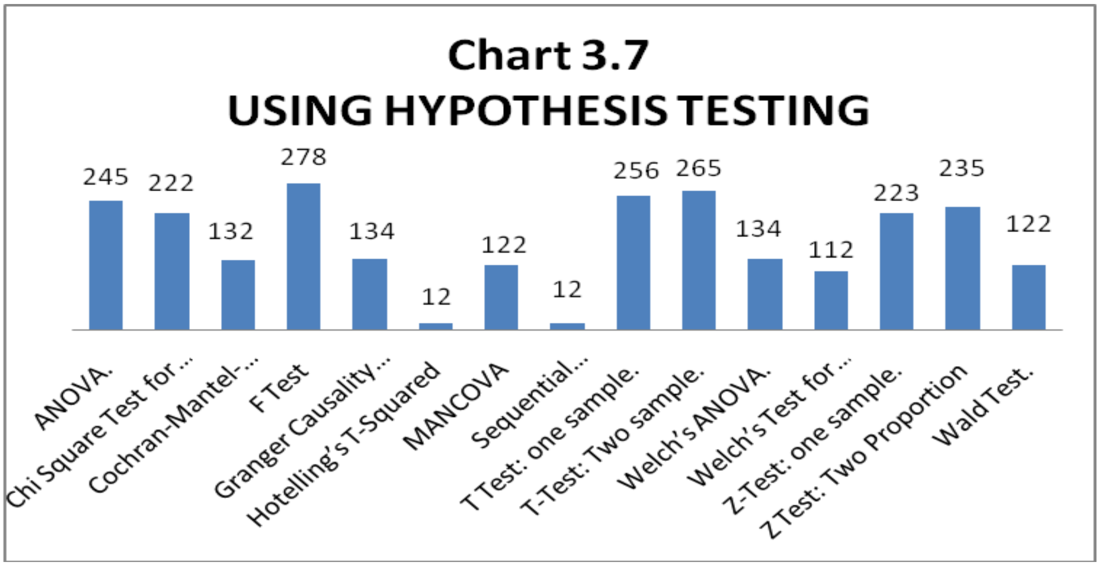
Correlation finds the degree of relationship between the variable compare to any to a variable to find the degree of relationship and regression help to find the unknown variable from the known variable help to budgeting and forecasting.



28% of the respondents like to use Karl Pearson coefficient and 26% of respondents use spearman's rank

USING HYPOTHESIS TESTING

Find the significant relationship between the variable the good social science research needs hypothesis analysis the relationship of collected data.



278 respondents are like to use the F- Test, 265 respondents have been used T-Test two-sample, and 256 respondents using T-Test one sample for the research works.

RESULT AND DISCUSSION

Chart 3.1 clear that, at the time of data collecting the primary date has been affected by the purpose of research and it has been limited the thing of the investigators.

Chart 3.2 expresses that 41% of the researches prefer to use convenience sampling design

Chart 3.3 indicates that 60% of researches has been used median and arithmetic mean

Chart 3.4 shows that 29% of researches has been used mean deviation

Chart 3.5 clearly shows that 46% of researchers have been used by Karl Pearson co-efficient

Chart 3.6 indicates that researches 54% of have been used Karl Pearson coefficient and spearman's rank

The chat 3.7 has been showing that 278 respondents are like to use F- Test, 265 respondents have been used T-Test two-sample and 256 respondents using T-Test one sample

Limitation of the study

The study connected through Google forms, hence the investigator unable to observe the respondent feeling. Census method is the best sampling for this study for time and finance and also COVID -19 problems we fore to do convenient sampling methods.

SUMMARY

The finding of the study indicates that most of the researchers have been used the same tools for analyzing the data we give suggestions as to use different statistical tools as per the research needs and also sampling techniques must be improved.

9. APPLICATION OF MATHEMATICS IN DAILY LIFE

Aishwarya Anil Zele, A/p, Bhoj, Tal: Nipani, Dist; Belagavi

Abstract

Mathematics is applied everywhere. Literally, mathematics prevents chaos to make our life hassle free. It is fair to say that inheritance of mathematics inculcates qualities in human being that nurtures the power of reasoning, critical, creative and innovative thinking. It is the mathematics, which provides abstraction, spatial and modelling of complex problems to find and analyse the solutions. Further, communication becomes better and powerful due the application mathematics. Human being is not only the creature, which applies mathematics. There are countless examples around us formation of various mathematical patterns in nature's fabrication. Birds flying patterns, animals' movements and formation of roots, trunks, twigs and leaves of trees has their well-established mathematical relationships. Snails make their shells; spiders design their webs and bees build hexagonal comps are a few to strengthen our belief that even insects use mathematics in their everyday life for existence. In our day-to-day life whatever role, we play be it a cook or a farmer, a carpenter or a mechanic, a shopkeeper or a doctor, an engineer or scientist, a musician or a magician, every one of us needs mathematics. Hence, it is quite impossible to summarize the applications of mathematics even in a single field. Through this paper, it is intended to talk about the application of mod theory in our everyday life. How mathematics used in the fault tolerant systems to avoid hazards, accidents in industries by checking and correcting the errors. How mathematics helps in building reliable and dependable industrial systems.

Introduction

Mathematics is a methodical application of matter. It is so said because the subject makes a man methodical or systematic. Mathematics makes our life orderly and prevents chaos. Certain qualities that are nurtured by mathematics are power of reasoning, creativity, abstract or spatial thinking, critical thinking, problem solving ability and even effective communication skills. Mathematics is the cradle of all creations. Be it a cook or a former, a carpenter or a mechanic, a shopkeeper or a doctor, an engineer or a scientist, a musician or a magician, everyone needs mathematics in their day-to-day life. Even insects use mathematics in their everyday life for existence. Snails make their shells, spiders design their webs, and bees build hexagonal combs. There are countless examples of mathematical patterns in the nature's fabric. It is a tool in our hands to make our life simpler and easies the present age is one of skill-development and innovations. The more mathematical we are in approach, the more successful we will be. During ancient times, the Egyptian used maths and complex mathematic equations like geometry and algebra. Maths is a part of our lives, whether we clean the house, make support or mow the lawn. Math has become an inseparable part of our lives and whether we work in an office or spend most of our time at home, each one of us uses math as a part of our everyday life no matter where we are as well as whatever we are doing, math is always there where you notice it or not. Mathematics is a universal language in that it is a language understood in all countries.

History of mathematics

- The area of study is known as history of mathematics is primarily discovered an investigation into the origin of discoveries in mathematics and to a lesser extend an investigation into the mathematical methods and notation of the past.
- The study of the mathematics as a study in its own right begins in the 6^{th} century BC with the Pythagoras, who coined the term 'mathematics' from the ancient Greek (meaning mathema), meaning "subject of instruction" Greek mathematics greatly refined the methods and expanded the subject matter of mathematics.
- Beginning in Renaissance Italy in the 16^{th} century, new mathematical developments, interacting with new scientific discoveries, were made at an increasing pace that continues through the present day.

Importance of mathematics in our life

- Consider a man going for groceries shopping. He would need to add up the total value of the things he bought and needs to calculate how much change he should ask for back. All that involves mathematics.
- It is also useful in calculating environmental statistics. It helps us to calculate the environment conditions namely monsoon arrival, earthquake forecasting, heavy rainfalls, tsunamis etc.
- Mathematics has many important practical applications in every facet of life, including computers, space exploration, engineering, physics, and economics and commerce.
- For example, mathematician knew about binary arithmetic, using only the digits 0 and 1, for years before this knowledge became practical in computers to describe switches that are either off (0) or on (1).
- The laws of probability led to our ability to predict behaviours of large populations by sampling.
- The importance of mathematics lies in its application in daily life. Its applications include in the fields of science, technology, economics, business, commerce and computer design,

- On a basic level you need to be able to count, multiply, subtract, and divide. It is present in different forms whenever we pick up the phone, manage the money, travel to some place, play soccer, meet new friends; unintentionally in all these things mathematics is involved

Applications

1. Cooking:

People use mathematics knowledge when cooking. For example, it is very common to use a half or double of a recipe. In this case, people use proportions and ratios to make correct calculations for each ingredient. If a recipe calls for 2/3 of a cup of flour, the cook has to calculate how much half is or double of 2/3 of a cup. Then the cook has to represent the amount using standard measures used in baking, such as 1/4 cup, 1/3 cup or 1 cup.

2. Gardening:

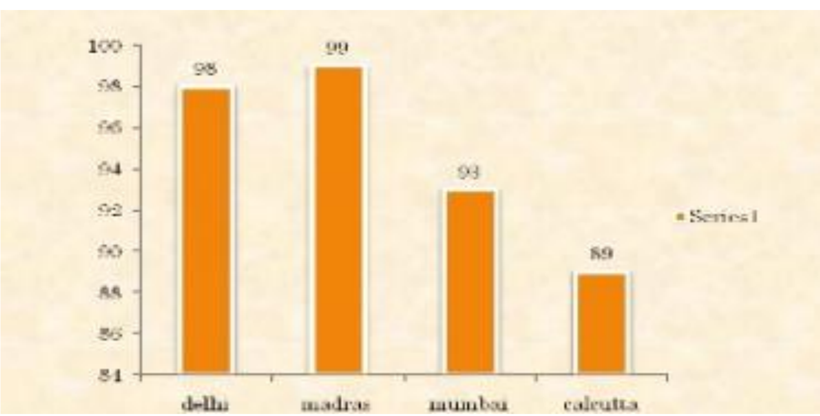
- Even doing something as mundane as gardening requires a basic math skill.
- If we need to plant or sow new seeds or seedlings you need to make a row or count them out or even make holes.
- Measuring skills is always needed, and calculations of the essence when doing something new in garden.
- Sometimes we have to calculate the area of the field for sowing seeds.

3. Banking:

- In bank people deposit their money and lend money from banks for any purpose, the interest is calculated on the basis of mathematical formulas.
- With some knowledge of simple and compound interest we can grow our money.
- Banking is probably the one place where math is used more than any where else. Going to bank means that you need to have proper accounts of the money that you want to withdraw, deposit etc.

4. Statistics:

- Statistics is a branch of mathematics which deals with data and interpret it to give us valuable information about the output.
- For example – the survey of India maintains the kinds of statistic data including population, land area, literacy rate etc. with the use of mathematics we are able to calculate future trends of variable data and predict about future.



5. Area:

- In our day to day life we have to calculate the area of any geometrical shapes like when we whitewash our buildings, purchase a land or for construction purposes etc. If you want to calculate how much paint, wallpaper, flooring, carpeting or tile you have to buy for your project you must know the area of wall and floor.

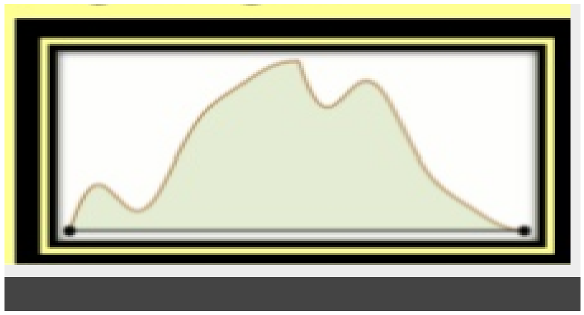
6. Sports:

- The mathematical concept can be applied when one wants to comment about the game.
- When we toss in game, probability is used as there is an equal chance of getting head or tail.
- Reference and coaches make calls which are based on time.
- We can compare failure and success rate by collecting data of all the past game played. This can only be done using some aspects of maths.
- To make the field of correct measure, we need good knowledge of mensuration.

7. Finding distance and length:

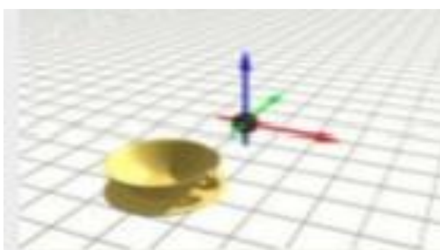
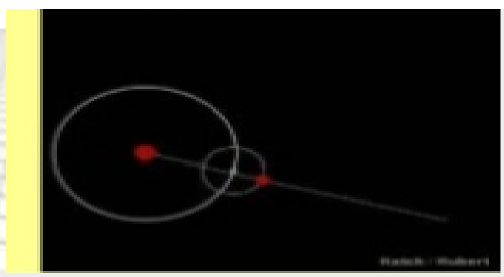
   Maths concepts are used to calculate –

- Height of towers, mountains and tides in ocean.
- Distance of the shore from a point in the sea and between celestial bodies.
- Structural load, roof slopes, ground surfaces and many other aspects, including sun shading and light angles.



8. Navigation:

- Concept is widely used while working with graphic designing software.
- We often use it to navigate a city or to locate objects in a grid.
- When geo catching, we use latitude and longitude.
- Astronomers (amateurs) use sky coordinates locate objects and interest.

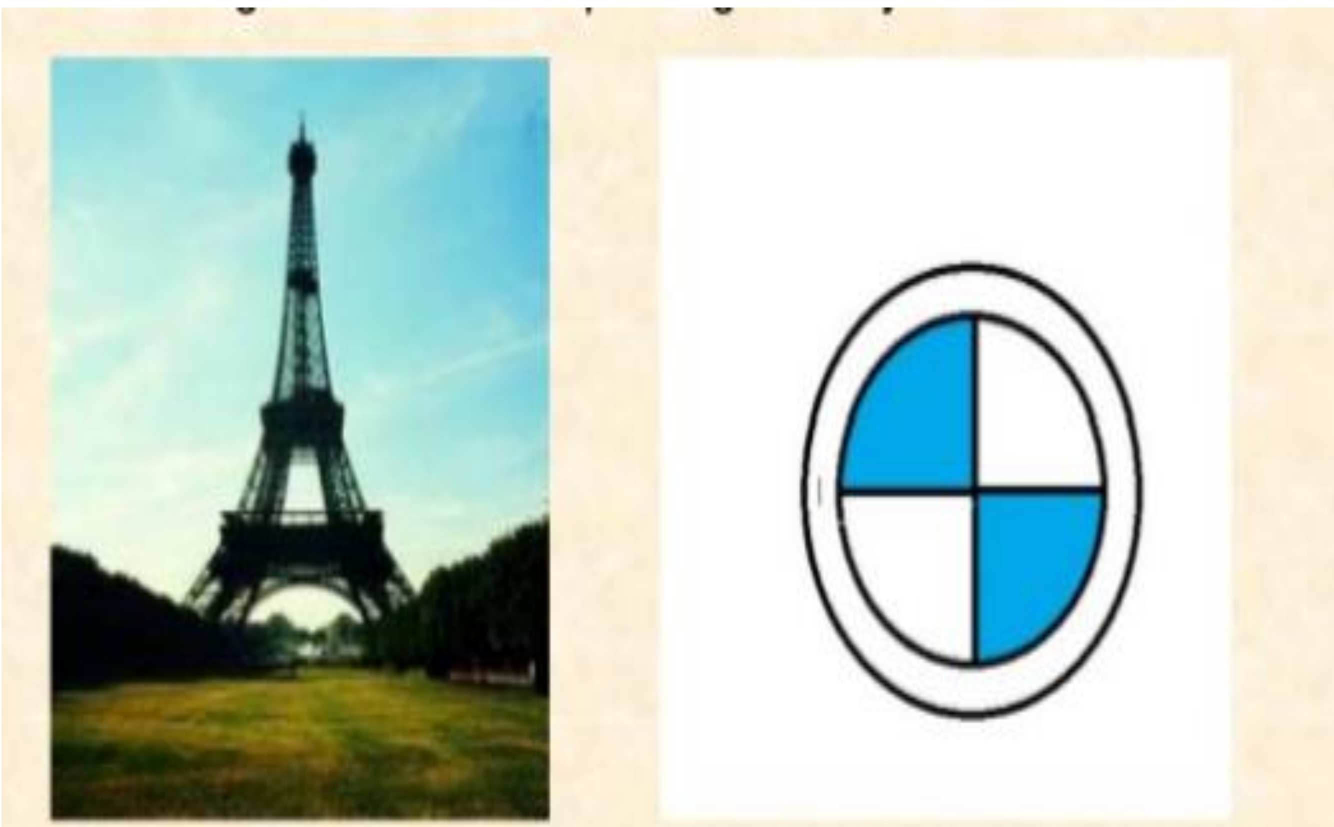
9. Arts and fashion:

   Math is also used by artists to draw geometric patterns on fashionable cloths. They also use math for calculating the amount and cost of fabric required. Artist also use geometry to design patterns for houses decorations and for clothes

10. Art and architecture:

- Architects use math to calculate the square footage of rooms and buildings, to lay out floor space dimensions and to calculate the required space for other areas such as parking, plumbing etc. The famous example of geometry in architecture is Eiffel tower. The famous car company BMW's logo is a fine example of geometry in art

- Symmetrical and geometrical concepts have been applied in the construction of some famous monuments such as Taj mahal, Eiffel tower etc.
- Areas of differently sized and shaped lands can be found through the application of area formulas.
- The Pythagoras theorem is used to measure of size for a painting a building or finding the shortest route to a particular location.
- A water tower and many other structures include many solid shapes whose volume can be calculated through geometry.



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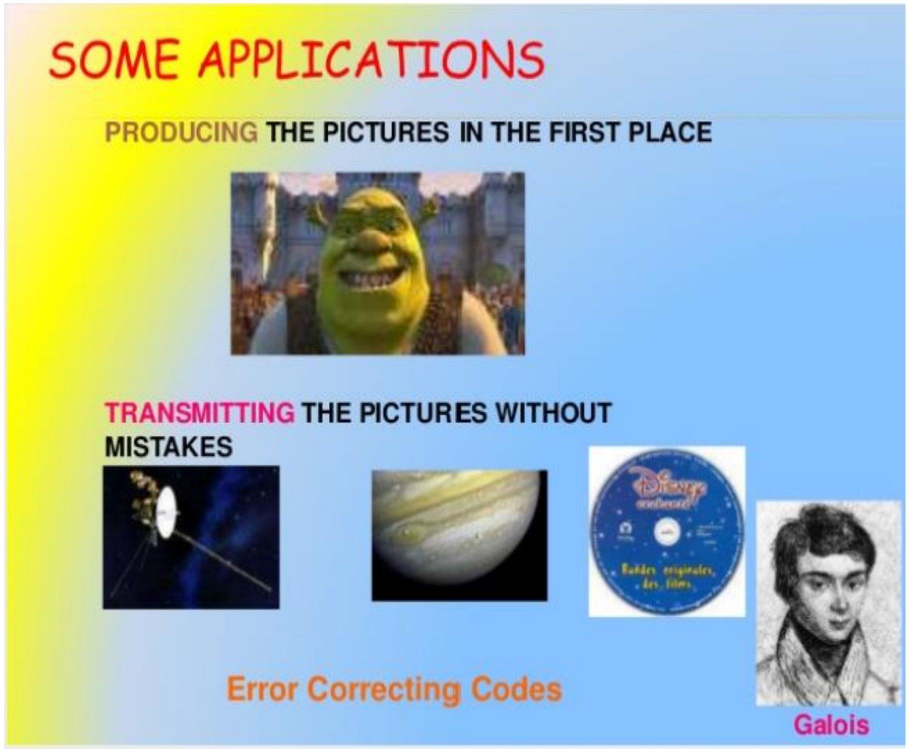
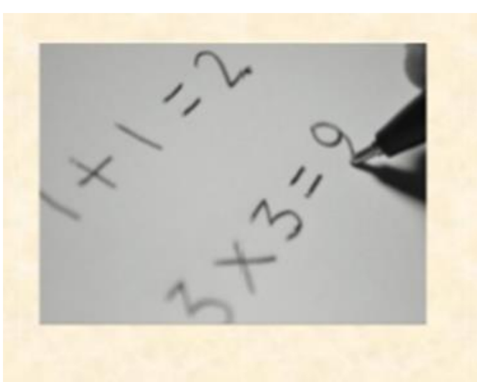
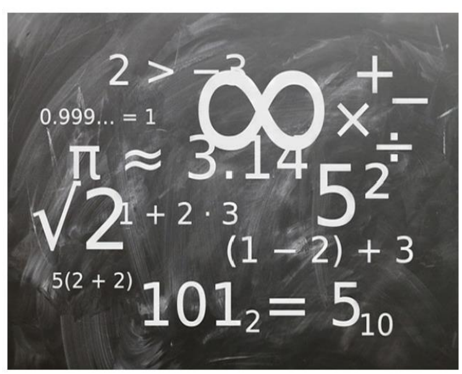


Conclusion

- We are surrounded by the math in each and every steps of our life we feel the necessity of math. It has wide application in our daily life, from the kindergarten to the graduation, from home to tuition everywhere we are surrounded by the application of math it has been became an integral part our life.
- The uses of mathematics in one's life is infinite. The use of mathematics is unquestionable for every individual. Being the queen of all sciences and the king of all arts, it offers a wonderful approach to us to be successful in life.





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10. DIFFERENTIAL EQUATIONS WITH MODELING APPLICATIONS

Anuja G. Patil., KLE Society's G I Bagewadi Arts, Science and Commerce College, Nipani.

Abstract: An equation containing the derivatives of one or more unknown functions with respect to one or more independent variables is said to be a differential equation. We can classify differential equations according to type, order and linearity. Based on type, differential equations can be classified as Ordinary Differential Equations (ODE) and Partial Differential Equations (PDE). Based on order differential equations can be classified as 1^{st} order differential equations, 2^{nd} order differential equations, 3^{rd} order differential equations and all higher order differential equations, the order depends upon the highest derivative present in the differential equation. Lastly, based upon linearity differential equations can be classified as linear differential equations and non-linear differential equations. If we consider modelling applications of differential equations we may come across some models in biology, chemistry and physics. Mathematical models are often desirable to describe the behaviour of some real life systems, whether physical, sociological or even economical in mathematical terms. For example, we may wish to understand mechanism of certain ecosystem by studying the growth of animal population in that system, or we may wish to date fossils by analysing the decay of a radioactive substance, either in the fossil or in the stratum in which it was discovered. In case of differential equations, a single differential equation can serve as a mathematical model for many differential equation.

Introduction: In mathematics, the history of differential equations traces the development of "differential equations" from calculus, which itself was independently invented by English physicist Isaac Newton and German mathematician Gottfried Leibniz. Differential equations began with Leibniz, Bernoulli brothers and others from the 1680s, not long after Isaac Newton's 'fluxional equations' in 1670s.

Differential Equations: A differential equation is an equation containing the derivative of one or more dependent variables with respect to one or more independent variables.

For example:

$$\frac{dy}{dx} = 2xy \qquad \frac{xdy}{dx} = y - 1 \qquad \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$$\frac{\text{đ}u}{\text{đ}y} = -\frac{\text{đ}v}{\text{đ}x}\frac{\text{đ}x}{\text{đ}y} + \frac{\text{đ}y}{\text{đ}x} + 2 = 0 \qquad \frac{\text{đ}^2u}{\text{đ}x^2} + \frac{\text{đ}^2u}{\text{đ}t^2} = 2\frac{\text{đ}u}{\text{đ}t}$$

Types of differential equations:

1) **Ordinary Differential Equations (ODE):** An equation contains only ordinary derivatives of one or more dependent variables of a single independent variable.

   For example:

$$\frac{dy}{dx} + 5y = e^x \frac{dx}{dt} + \frac{dy}{dt} = 2x + y$$

2) **Partial Differential Equations (PDE):** An equation contains partial derivatives of one or more dependent variables of two or more independent variables.

For example:

$$\frac{đ^2 u}{đx^2} + \frac{đ^2 u}{đt^2} = 2\frac{đu}{đt}\frac{đu}{đy} = -\frac{đv}{đx}$$

Applications of Differential equations: The differential equations have wide applications in various engineering and science disciplines. In general, modelling variations of physical quantities such as, temperature, pressure, displacement, velocity, stress, strain, or concentration of a pollutant, with the change in time t or location such as coordinates (x, y, z) or both would require differential equations. Similarly, studying the variation of a physical quantity on other physical quantities would lead to differential equations. For example, the change of strain over stress for some viscoelastic material follows a differential equation.

Modelling applications of ODE:

1. Newton's law of cooling.
2. Electrical Circuits.
3. Population growth and decay.
4. Radioactive half-life.
5. Newton's 2^{nd} law of motion.
6. Physical applications.

1. Newton's law of cooling: The rate of change of the temperature of an object is proportional to the difference between it's own temperature and the temperature of its surroundings.

$$\text{i.e.}\frac{d\phi}{dt} = E\ A\ (\emptyset a - \emptyset t)$$

where, E= Constant that depends upon the surface area A, Øa- certain temperature, Øt- room temperature or the temperature of the surroundings. Therefore, Newton's law of cooling is an example for modelling applications of differential equations and it can be used to determine the time of death, to calculate the surface area of an object, etc.

2. Electrical series R-L circuit: Let a series circuit contains only a resistor and an inductor. By Kirchhoff's 2nd law the sum of the voltage drop across the inductor ($L\frac{di}{dt}$) and the voltage drop across the resistor (*iR* is same as impressed voltage (*E (t))* on the circuit. Current at time t, i(t), is the solution of the differential equation,

$$Ri + L\frac{di}{dt} = E$$

Thus, differential equations are often used in electrical circuits which we are familiar in day today life. Hence, it is also a modelling application of differential equations.

3. Population growth and decay: The nature of population growth and decay can be explained using below equation;

$$\text{i.e.}\frac{dN(t)}{dt} = k\,N(t)$$

Where, N(t) denotes population at time t and k is a constant of proportionality, serves as a model for population growth and decay of insects, animals and human population at certain places and duration. Solution of the above differential equation can be given:

i.e.$\frac{dN(t)}{dt} = k\,N(t)$

Integrate above equation on both sides we get,

$lnN(t) = kt + lnC$

Where, C is the constant of integration.

⇨ $ln\frac{N(t)}{C} = kt$

⇨ $N(t) = Ce^{kt}$

C can be determined if N(t) is given at certain time.

4. Radioactive half decay: It is a random process. The equation is given as,

$$\frac{dN}{dt} = -kN$$

The rate of decay depends upon the number of molecules or atoms that are there.

The negative value is due to the number is decreasing. K is constant of proportionality.

5. Newton's 2nd law of motion: The rate of change in momentum encountered by a moving object is equal to the net force applied to it.I n Mathematical terms, $F = \frac{d(mv)}{dt}$

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$F = ma$$

6. Physical applications: Some other simple modelling applications of differential equations are, velocity and acceleration. i.e. velocity, $V = \frac{ds}{dt}$and acceleration, $a = \frac{dV}{dt}$

Modelling applications of PDE:

1. Laplace equations.
2. Heat equations.
3. Wave equations.

1. Laplace equations: The Laplace equation is used to describe the steady state distribution of heat in a body. Also is used to describe the steady state distribution of electrical charge in a body.

$$\frac{\partial^2 u(x,y,z)}{\partial x^2} + \frac{\partial^2 u(x,y,z)}{\partial y^2} + \frac{\partial^2 u(x,y,z)}{\partial z^2} = 0$$

2. Heat equations: PDE is used to represent the heat equation in following form,

$$\frac{đu(x,y,z,t)}{đt} = œ\left(\frac{đ^2u}{đx^2} + \frac{đ^2u}{đy^2} + \frac{đ^2u}{đz^2}\right)$$

The function u(x, y, z, t) is used to represent the temperature at time t in a physical body at a point with coordinates (x, y, z).And œ is the thermal diffusivity. It is sufficient to consider the case œ=1.

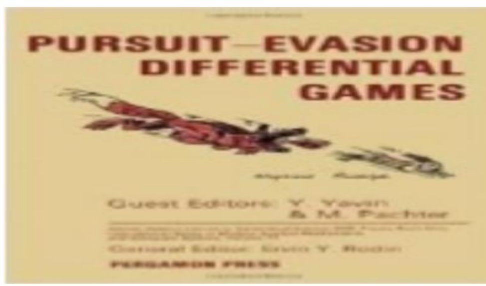
3. Wave equations: The wave equation can be represented in the form of PDE as follows:

$$\frac{đ^2u(x,y,z,t)}{đt^2} = c^2\left(\frac{đ^2u}{đx^2} + \frac{đ^2u}{đy^2} + \frac{đ^2u}{đz^2}\right)$$

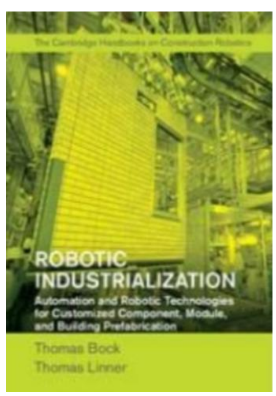
Where, the function u(x, y, z, t)is used to represent the displacement at time t of a particle whose position at rest is (x, y, z).And, constant c represents propagation speed of the wave.

Some other modelling applications of differential equations:

- Game theoretic models, building block concept and many applications are solved with differential



  equations.
- Graphical interference of analysing data and creating browser based on partial differential equation solving with finite element method.
- Auto motion and robotic technologies for customized component, module and building.



Conclusion: Hence, the differential equation models are used in many fields of applied physical science to describe the dynamic aspects of systems. The typical dynamic variable is time, and if it is the only dynamic variable, then the analysis will be based on Ordinary Differential Equations (ODE) model. When, in addition to time, geometrical considerations are also important, Partial Differential Equations (PDE) models are used. PDE models together with their boundary and initial conditions, arguably constitute the most sophisticated and challenging models in today's science.

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11. A STUDY ON IMPACT OF COVID-19 ON INDIAN ECONOMIC GROWTH (IN THE PRESENT CONTEXT)

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ABSTRACT

Indian Economy is the fastest developing economy after China with average GDP (Gross Domestic Product) around 7 percent, from (2014 - 2019) and can be said as one of the top markets for MNC`s and for developed economies for selling their consumer goods in Indian markets, especially there is lot of demand for technology, modern smart phones, laptops, pesticides. Agriculture contribution was the backbone of the India`s growth after independence, till 1991AD, after LPG (Liberalisation Privatisation and Globalisation) many sectors like service sector, manufacturing and some sectors contributed to growth of the Indian Economy. Due to the covid-19 pandemic, there is imbalance in market demand and supply in the Indian economy. It has affected India from March 24th 2020 of Indian Government lockdown, most of the public and private sectors, seaports firms, Industries and service sectors were closed in India due to lockdown of Central government and state governments. Some organised companies provided work from home option, but in India more than 75 percent of the working population are working in unorganised sectors, due to deficiency in technological equipment's or technological knowledge made unorganised sectors workers to decrease in the standard of living, with increase in unemployment ratio, and unavailability of proper trade facilities has increased the burden of farmers to transport crops yield to the Markets or mandi's (in rural areas). According to the report of (CMIE) "Centre of Monitoring Indian Economy", unemployment rate shot up significantly from 7.87 percent in June 2020 to 23.48 percent in May 2020, due to effect of Covid-19.

The impact of Covid-19 has resulted three major factors for economic crises in India, First one is decrease in GDP or National Income (NI), due to imbalance in demand and supply, lack of trade and transport facilities, decrease in demand for oil and natural gas. Second one is increase in unemployment ratio, according to many reports like CMIE, ADB (Asian Development Bank) the unemployment ratio in India rose above 20 percent after 1930 great depression. Third one is the health emergency in India and the world where more than 90 percent of the world has been effected by Covid-19 and unable to find specific medicine in the world which has led to economic crisis. The purpose of the study is to study the impact of Covid-19 on Indian economy,

which has led to economic crisis and the fiscal and monetary policies which can be used by the government and central bank of the country, to overcome economic crisis in Indian economy.

KEY WORDS: covid - 19, national income (GDP), employment, economic crisis and Indian economy.

INTRODUCTION:

Covid-19 is one of the global epidemic, was detected first time in China's Wuhan city (Hubei province) on 31.12.2019, from December to end to one month only few cases infected with pneumonia of unknown cause was found, these made china to declare National emergency in most of its parts, by intimating to WHO (World Health Organisation), WHO named Corona virus as COVID-19, and declared as public health emergency on 30th January 2020 in the world. From February 2020 Covid-19 affected most of the countries in the world. Almost more than 44, 42,414 cases were conformed, with 2,98,322 conformed death cases of Covid-19 around the, India has a greater challenge during and post Covid-19 situation first one is to cope up in covid-19 situation, because pandemic has affected almost all the states in India and the world, so resulted in lockdown of states in India , second one is to overcome the economic crisis in India, due to imbalance in demand and supply, most of the sectors in the Indian economy is unable to produce and contribute taxes to the Indian Government. Third one is unemployment problem in India, and migrants are not able to get employment in cities of India, major metro cities Delhi, Mumbai and Bangalore, according to the data published by the CMIE (Centre for Monitoring Indian Economy) digital inclusion in making nation into-self-reliant India during and post covid-19 situation.

After the spread of Covid-19 from China many countries have shut down their sea ports and airports, and stopped exports and imports. India, China is the major distributor of the raw materials to Europe and western nations. Which affect the manufacturing activities across the globe due to Government lockdowns. India is one the major developing country in Asia and the world, due to the pandemic the India government has preferred the health emergency by implementing lockdown in two phases, which affected the manufacturing activities and majorly it affects the supply chains of the country's economy. Indian economy has major task to overcome imbalance of supply chains in India, which has led to unemployment and economic crisis in India. National income is estimated to be low den the past decades. Indian economy can try to narrow the trade imbalance and try to improve in export of medical and pharmaceuticals, agriculture, fishing and other products, which has great economic demand in the world market, presently there is more demand for medical and pharmaceuticals, due to covid-19 pandemic in India and the world. India has major advantage in contribution of service sectors. A comprehensive strategy addressing the impact of the current crisis may put the Indian economy back on a sustained economic growth path and strengthen countries trade and foreign trade policy

II REVIEW OF LITERATURE

WHO

"A pandemic is a worldwide spread of new disease. An influenza pandemic occurs when a new influenza virus emerges and spread around the world and most of the people do not have immunity".

Adam Smith

Economics of the time were dominated by the idea that a country's wealth was best measured by its store of gold and silver. Smith proposed that a nation's wealth should be judged not by this metric but by the total of its production and commerce—today known as the gross domestic product (GDP)

UC CDC Definition

Pandemic refers to an epidemic that has spread over several country or continents affecting large number of people.

III NEED OF THE STUDY

1. The outbreak of the covid-19 is posing a challenge to many Business and economic activities and labour sections in India and the world.
2. To understand the problems faced by migrant workers during covid-19 pandemic.
3. To understand the policy and packages by the government to overcome unemployment in world, with government rules and regulations.
4. A business strategy that involves selling products and services in different foreign markets without changing the characteristics of the product/service to accommodate the cultural norms or customs of the various markets.

IV STATEMENT OF THE PROBLEM

Overall Covid-19 has brought uncertainty to the Indian Economy and a large section of low income of population around the globe. Especially low-income individuals, workers or migrants are worst hit by this pandemic in India, due to 40 day's lockdown of the Indian Government and its states. The uncertainty about future looms heavily in the mind of both consumer and producers. Resulted in decrease in growth of the Economy, slowdown in National Income (NI), causing economic crisis, in many states in India. But the concerted action by the countries in the world will surely turn the tide. India and some other countries which has advantage in production in pharmacy and other manpower resources, have great opportunities in this circumstance especially looking at the composition of global value chains in the world trade with surplus manpower, but the problem of increase in disequilibrium of payment, sudden increase and decrease in price, monopoly market of some powerful MNC`s (Multi-National Companies) in developing countries has made these countries to depend more on other Developed economies for import of technology and investment.

V OBJECTIVES OF THE STUDY

1. To understand government fiscal policies, packages to overcome the problem of health crises, with suitable fiscal policies to achieving India's goal Self Reliant India or Atmanirbhar Bharat
2. ..To enable substantial growth of production, labour, national income and service sector in national and international trade.
3. Its effects national income, flexible trade, good business, economic crisis and balance of payment.
4. To understand employment, inclusive growth and security for unorganised sector in the economy.

VI RESEARCH METHODOLOGY

This research output is the outcome of an overview conducted on the impact of covid-19 on Indian Economy, migrant workers and international business, Economic crisis and trade in the Indian context experiential approach. It uses secondary data for analysis, discussion with experts from part of the research work.

i NATURE OF STUDY

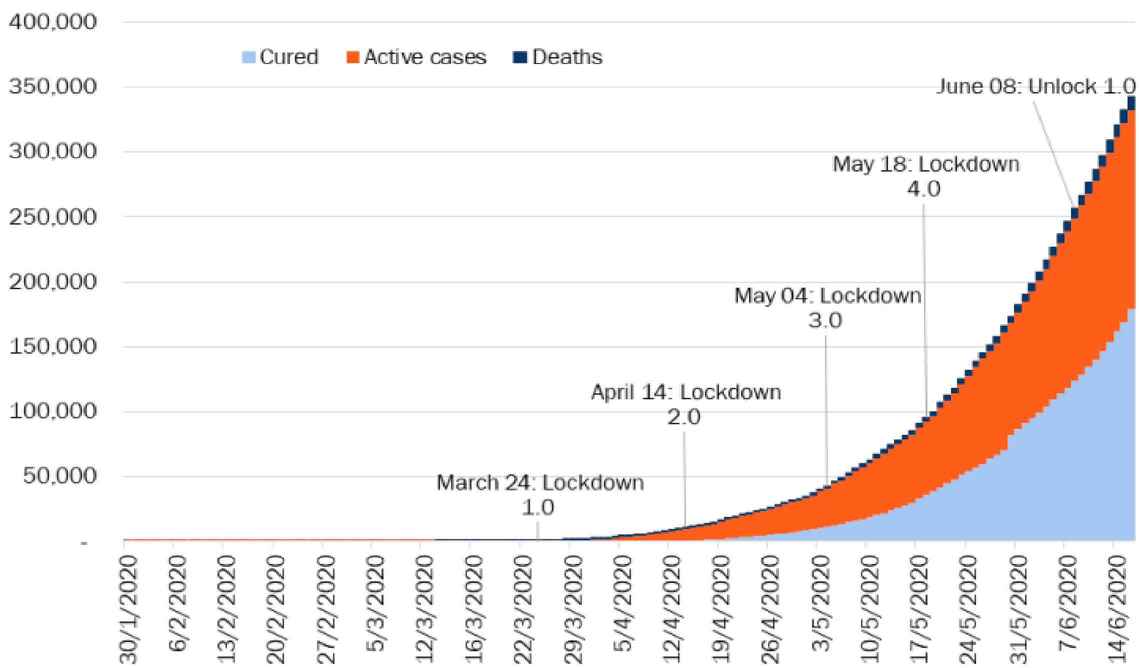
The study is mainly descriptive in nature; secondary data are used for the purposes of the study.

ii SECONDARY DATA

Secondary data was collected from websites, various articles and journals

DATA AND STATISTICS

Figure 1. Total (cumulative) number of cases of COVID-19 in India



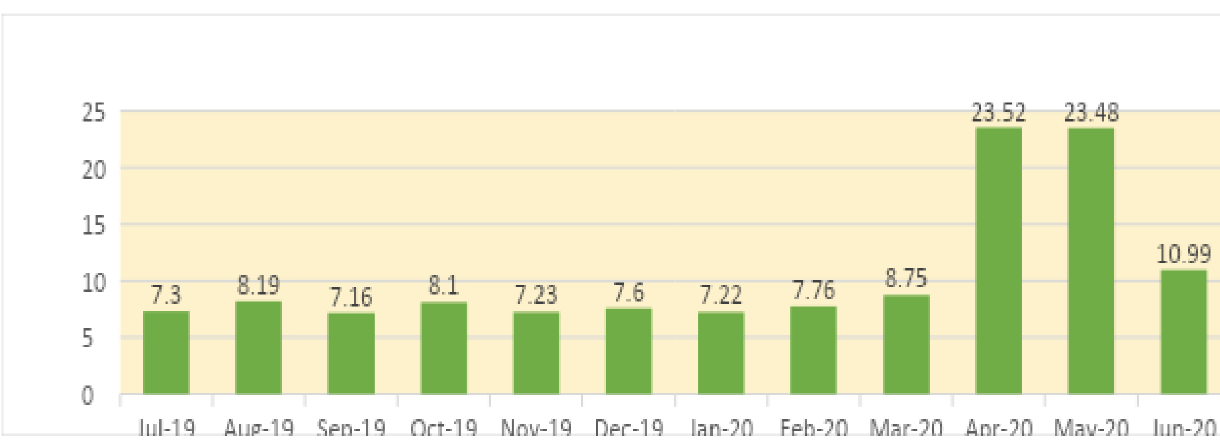
Source: Ministry of Health and Family Welfare, Government of India. BROOKINGS

i

The graph indicates that most of the Economic activities in India was almost standstill, this resulted in low demand for petroleum products, oil, sales and business activities, but after lockdown 3 and 4 there was some concision given to some development and economic activities, unlock phase after June 8 resulted in drastic increase in covid-19 cases, but resuming economic activities in India.

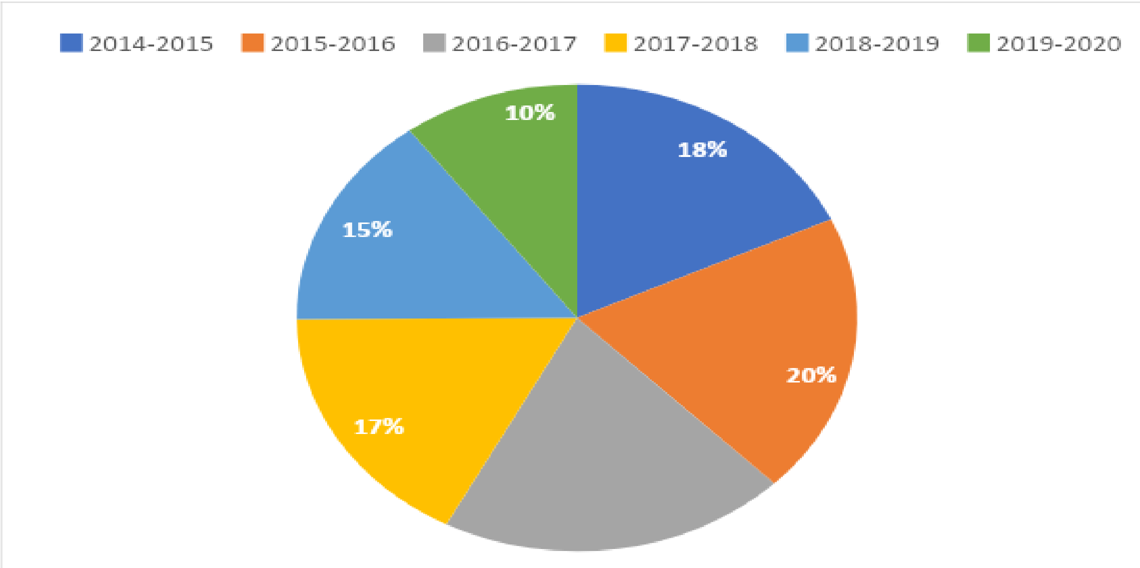
ii

INDIAN UNEMPLOYMENT RATE BACK DOWN AFTER COVID-19



The above graph explains the increase in unemployment percentage during Covid-19 pandemic from July 2019 to June 2020, as most of the working population in India works in unorganised sector, there is sudden rise in unemployment from March 2020.

iii India’s GDP Growth Hits New Low



Above graph signifies the decrease in GDP (Gross Domestic Product) in India from year 2014 to 2020, where the contribution of all sectors of GDP is unable to contribute accordingly in year 2020.data showsnegative impact of Covid-19 on Indian Economy.

VII RESULT AND DISSCUSION

1. Global pandemic Covid-19 which initially made China to suffer health and economic losses, now it has its impact on India and most parts of the world, from March 2020 India’s GDP ,world GDP has been hit hard, resulted in increase in unemployment, decrease in demand in India and globally. Government and international agencies has to take care to resuming economic and business activities, otherwise it may impact severely in a long run. It understands the need of sustainable growth of the economy, economic development by increase in HDI (Human Development Index), rise in PCI (per Capita Income) in Macro or aggregate view.

2. **VIII FINDINGS**

1 Covid-19 has emerged as a bigger challenge for the globe to resume economic and trade related activities for flexible Economic growth.

2 Most of the national, international movement of labour has led to increase in unemployment ratio of India, around 11percent in June 2020, according to the data published by (CMIE).

3 In India due to lack of sanitization and precautionary measures, migrant workers are facing hardship to sustain in metro cities, and they are finding hard to earn their daily wages.

4 International trade may increase job opportunities, but due to governments' lockdown from March 2020, international trade related economic activities are persistent.

5 It is a tuff task for government to provide enormous digital facilities for unorganised sector in short span of time, like digitalisation, relief package in the pandemic situation.

IX RECOMMENDATIONS \SUGGESTION

1 Promotion of MSME (Micro Small and Medium Enterprises) which contributes around 29 percent in GDP and digital India to all economic, financial sectors of the country.

2 Flexible tax evaluation for all section of society, including relief packages to economically weaker sections of the society during covid19, to reach India's goal "Atmanirbhar Bharat or Self Reliant India".

3 Fiscal and monetary policies implemented according for the flexible demand and supply of labour, capital in a country accordingly.

4 India has to implement properly government schemes like "Make in India and Digital India", because India still use most of the foreign developed technology in some sensitive fields like defence, security.

5 Macroeconomic policies like Swatch Barat Mission have to be implemented to provide job security for migrant workers, unorganised sectors and below poverty line individuals.

X CONCLUSION

International trade includes new economic, foreign policies implemented by the government, EXIM policy, and it creates employment to large section of population. Indian foreign trade which has come to stand still due to unavailability of workers at airports, seaports as many trade sectors are finding hard to export and import goods and services due to the pandemic. Indian workers who migrated to foreign countries for better job facilities and to increase in standard of living, most of them have returned to India due to covid-19, India has a huge task to invest in job providing sectors like MSME, Every country has limited resources therefore a country cannot produce all the goods and services that it requires, due to some trade benefit factors like comparative advantage, availability of labour, technology, capital resource, land and required goods which cannot be produced or the amount is insufficient as require, needed to be provided from other countries similarly, countries sell their products to others also when the production of goods comes in surplus quantities than demanded in the country. International trade has led to growth of various sectors in India and the world, especially after 1991 LPG (Liberalisation, Privatisation, Globalisation), various industries likes automobiles and increase in inflow of FDI (Foreign Direct Investment) and services, this growth led to increase in competition of world market, but due to effect of covid-19, most of the countries international trades exports and imports has been decreased, due to unavailability of labour and border restrictions. Covid-19 has resulted global economic crisis, from March 2020 it has resulted in negative economic growth in Indian Economy, increase in unemployment, decrease in standard of living and problem of migrant workers. Prime Minister Narendra Modi has launched Make in India, initiative on September 25, 2014, with the primary goal of making India a global manufacturing hub by encouraging both multinational as well as domestic companies to manufacture their product in the country. Trade is central idea to ending global poverty, unemployment; International trade aim is to increase employment ratio and global inclusion.

XI LIMITATION

1) Covid-19 has led to global economic crisis and can create imbalance in demand and supply of labour in the Indian economy and the world.

2) Trade can be led to over specialisation with workers at work of losing their jobs when domestic goods are not exported due to covid-19 restrictions, may lead to increase in disequilibrium of balance of payments in the economy.

3) It has resulted in country dual economics in underdevelopment countries, as a result of inflexible trade and labour policies by the government, where migrant labourers or casual workers are vulnerable led to economic crisis.

4) Governments plans and programmes have to reach all the sectors of the economy, especially the vulnerable groups in India.

XII SCOPE OF THE STUDY

India's major export items are related to agricultural and informal sector activities, whose export demand also has come down, But this depressing situation is not expected to continue for long period of time, though it is not certain how long this pandemic will force the countries to keep all such activities under lock. Items of daily need have to be either produced or imported by all countries once the stock is over. In this respect India can go for diversification of some products depending on its expertise, especially in medical items, whose demand has got a sudden peak up in international market. And try to depend more on domestic production, increase exports may lead to create employment to large unorganised migrants or domestic labourers of the country, flexible fiscal and monetary policies may try to narrow the trade imbalance, increase in GDP growth, development of all the sectors of the economy providing job security.

XIII REFERENCE

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Covid-19 challenge for Indian Economy Trade and Foreign Policy Effect

Centre for monitoring Indian economy (CMIE), Bombay

APPENDIX

I INTRODUCTION

II REVIEW OF LITERATURE

III NEED OF THE STUDY

IV STATEMENT OF THE PROBLEM

V OBJECTIVES OF THE STUDY

VI RESEARCH METHODOLOGY

VII RESULT AND DISSCUSION

VIII FINDINGS

IX RECOMMENDATIONS \SUGGESTION

X CONCLUSION

XI LIMITATION

XII SCOPE OF THE STUDY

XIII REFRENCE

12. STUDY OF TOPOLOGY ON A GROUP

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Abstract: A prominent property of a group is that each element has inverse in it. Using this property we have studied a topology on a group G. Various topological properties of the topological space (G, τ) are studied with the help of different properties of the group G.

INTRODUCTION

According to Birkhofi [1] there is a great influence of Stones representation theorem on the subsequent developments of mathematics. The key idea that made Stones work, a genuinely new starting point, was the fact that topological spaces could be defined from purely algebraic systems such as Boolean algebra. The discovery that purely algebraic structures could give rise to topologically interesting spaces was literally revolutionary.

In this note we try to study a topology on a group, using the prominent property of a group that each element in the group has an inverse in it. This topological space has many interesting topological properties.

For the basic concepts in algebra we refer [2] and for the basic concepts in topology we refer [3]. Throughout the paper, by G, we mean the group $(G, +)$.

2 TOPOLOGY ON A GROUP

We define the topology on a group G in the following theorem.

Theorem 2.1 Let G be a group. Define

$\tau = \{\phi\} \cup \{A \subseteq G | x \in A \Longrightarrow -x \in A\}$

then τ is a topology on G, where -x is the inverse of x with respect to +.

Proof: (i) By definition $\phi \in \tau$. If $x \in G$, then $-x \in G$. Hence $G \in \tau$

(ii) Let $A; B \in \tau$. If $A \cap B = \phi$ then $A \cap B \in \tau$, Suppose $A \cap B \neq \phi$. Select $x \in A \cap B$,

Then $x \in A \cap B \Longrightarrow x \in A \; and \; x \in B$

$\Longrightarrow -x \in A \; and \; -x \in B \; \ldots$ (By definition of τ)

$\Longrightarrow -x \in A \cap B$

$\Longrightarrow A \cap B \in \tau$.

(iii) Let $A_\lambda \in \tau \;\; \forall \lambda \in I$,where *I* is any indexing set. Then

If $\cup A_\lambda = \phi$, then $\cup A_\lambda \in \tau$

If $\cup A_\lambda \neq \phi$, select $x \in \cup A_\lambda$.

$x \in \cup A_\lambda \Rightarrow x \in A_{\lambda_0}$, for some $\lambda_0 \in I$,

$\Rightarrow -x \in A_{\lambda_0}, \ldots$, (as $A_{\lambda_0} \in \tau$)

$\Rightarrow -x \in \cup A_{\lambda_0}$.

Thus from (i),(ii) and (iii) we see that τ is topology on G.

Corollary 2.1*If* $A \in \tau$ *then* $G - A \in \tau$, *where* $G - A$ *is complement of A in G.*

Proof: Let $x \in G - A$.

Then $x \notin A \Longrightarrow -x \notin A \Rightarrow -x \in G - A$.

Thus for each $x \in G - A$ we get $\Rightarrow -x \in G - A$.

Hence $G - A \in \tau$.

Corollary 2.2 If *2x = e* for every *x* in *G* (if each element in a group is its own inverse), then τ is discrete topology on G.

Proof: $2x = e \;\forall x \in G$ gives $x = -x \;\; \forall x \in G$.Thus we get $\{x\} \in \tau, \;\forall x \in G \Longrightarrow \tau = \wp(G)$.

Hence τ is the discrete topology on G.

Remark 2.1 (i) Every member of τ is clo-open.

(ii) If *H* is a subgroup of *G*, then H $\in \tau$.

(iii) Define $y = \{x \in G | x = -x\}$ then the relative topology τ^* on Y is a discrete topology.

The relative topology τ^* on Y is given by $\tau^* = \{A \cap Y | A \in \tau\}$

As $x = -x \;\; \forall x \in Y$, we get $\{x\} \in \tau^* \;\; \forall x \in Y$.

Hence τ^* is the discrete topology on Y.

(iv) The family $\mathfrak{B} \subseteq \tau$ defined by

$\mathfrak{B} = \{\{x\} | x \in G \ and \ x = -x\} \cup \{\{x, -x\} | x \in G \ and \ x \neq -x\}$ is a base for the topology τ.

(v) If *O(G)* = *2n*, then there exist even number of elements in G for which $x = -x$. Hence for the group of even order the number of singleton sets in τ is also even.

(vi) For each *x* in *G* $\{\{x, \ -x\}\}$ or $\{\{x\}\}$ will form a countable local base depending on $x \neq -x$ or $x = -x$ respectively. Hence $(G, \ \tau)$ is a first axiom space.

(vii) If τ contains all singleton subsets of *G*, then *G* must be an abelian group, as $x = -x$ for each $x \in G$.

(viii) $(G, \ \tau)$ is not a T_0 space, in general.

Theorem 2.2 The space $(G, \ \tau)$ is both regular and normal.

Proof: Let F be a closed set and $x \notin F \Rightarrow x \in G - F$

As F is closed, $G - F$ is open.

Hence $-x \in G - F$ which gives $\{x, -x\} \cap F = \emptyset$. Further $F \in \tau$ (see remark 2.1 – (i)).

Thus for a closed set F and $x \notin F$, there exist an open set $\{x, -x\}$ containing x and an open set F with $\{x, -x\} \cap F = \emptyset$. This proves $(G, \ \tau)$ is a regular space.

As the member of τ are clo-open the space $(G, \ \tau)$ is a normal space.

We know that a topological space $(G, \ \tau)$ is disconnected if and only if there exist a proper clo-open subset in x.

Theorem 2.3 The space $(G, \ \tau)$ is a disconnected space iff $O(G) > 1$.

Proof: If $O(G) > 1$, then $\exists \ x \in G$ such that $x \notin e$.

If $x = -x$, then $\{x\} \subset G$ which is both open and closed.

If $x \neq -x$, then $\{x - x\} \subset G$ which is both open and closed.

Hence $(G, \ \tau)$ is not connected if $O(G) > 1$.

Remark 2.2 If $O(G) = 1$, then clearly G is connected.

For the well-known group $(\mathbb{Z}, +)$ we have

Theorem 2.4 For the space $(\mathbb{Z}, \ \tau)$ we have,

(i)The topology τ is not metrizable. (ii) $(\mathbb{Z}, \ \tau)$ is a second axiom space.

Proof: (i) Suppose $(\mathbb{Z}, \ \tau)$ is metrizable. Hence there exist a metric d on Z such that the induced topology τ_d by the metric d coincides with τ.

Now for $x \in Z, \{x\} = S(x,1) \in \tau_d \Rightarrow \{x\} \in \tau_d, \forall x \in Z$… (since $\tau_d = \tau$)

Thus $x = -x, \forall x \in Z$; which is not true. Hence our assumption is wrong. This proves that (Z, τ) is not metrizable.

(ii) Define $\mathfrak{B} = \{\{n, -n\} | n \in N \cup \{0\}\}$.

Then $\mathfrak{B}$ forms a countable base for τ. Hence $(Z, \tau)$is second axiom space. In general, (Z, τ) need not be second axiom space, e.g. for the group$(R, +)$, (R, τ) is not second axiom space.

A sufficient condition for the space (G, τ) to be second axiom (word) is given in the following theorem.

Theorem 2.5 If G is a cyclic group then (G, τ) is a second axiom space.

Proof: If G is a finite cyclic group then obviously it is a second axiom space.

If G is infinite cyclic group then $G \cong Z$. We know that (Z, τ) is a second axiom space (see theorem (2.4)). Hence $(G, \tau)$is second axiom space.

If A is a subgroup of G then $A \in \tau$. For converse part we have the following theorem.

Theorem 2.6 Let $A \neq \emptyset$ be a subset of G. $A \in \tau$ is a subgroup of G if and only if $a + b \in A, \forall a, b \in A$

Proof: Only if part is trivially true. For the if part suppose $a + b \in A, \forall a, b \in A$.

As $A \in \tau$, $-x \in A$ whenever $x \in A$. Hence $0 = x - x \in A$. Thus A is a subgroup of G.

The special property of the group $(Z, +)$ is that 0 is the only element of Z which is its own inverse. Such type of groups have the following property.

Theorem 2.7 The space$(G, \tau)$is countably compact, if *G* is an infinite group, with *e* as the only element in *G*, which is its own inverse.

Proof: Let E be any infinite subset of G. Then there exists $x \neq e$in *E*.

We prove $-x$is a limit point of E. Let A be any open set containing$-x$.

$A \in \tau$and$-x \in A$gives$x \in A$. Thus$x \in A \cap E$. Hence$x \in (A \cap E) - \{-x\}$, which shows that $(A \cap E) - \{-x\} \neq \phi$. This proves $-x$ is a limit point of E in G. Hence G is countably compact.

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13. The Growth of Smartphones using the Logistic Differential Equation

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Abstract: Mathematics has played a very important role in the human history; its importance is often undervalued. The changes that mathematics brought into the world are Groundbreaking like from the very first calculator to the enigma machine built by Alan Turing. The processing power of those computers were too small compared to the today's superior smartphone which uses numerous mathematical algorithms written in computer language to accomplish impeccable tasks in our daily life. Ever since the smartphone was invented most people have relied onto them for their tasks, but in knowing how many people in India are actively using their smart phones brings us insights like never before. To find the data, we use the simplest of the Differential equations of all which is the logistic differential equation. Using this Equation, a prediction can be made that the active smartphone users in India will rise to 700 million by the end of December 2020. With the data at hand the question is why do we need to know the active smartphone users in our country and what can be accomplished by it?

Keywords: Logistic Differential Equation; Pandemic; Smartphone Population; Opportunity; Growth;

1.1. Introduction Mathematics is the language in which God has written the universe"— Galileo Galilei .Mathematics has revolutionized the History and has played an integral part in it. Mathematics was used to identify telemetry of the very first flight by the Wright Brothers, crack the cryptic messages during the world war and much more. In recent times we use it most in a form of a computer which has been integrated to almost every electronic device made. These devices revolutionized the world and led us to the digital frontier. Mathematics is a vast subject and infinitely more applicable in today's modern world, of it Is one simple mathematical equation call the logistic differential equation, the equation is simple but the predictions that can be made using it are insightful and ever so easy to compute. The logistic equation has numerous fields of application like biomathematics, chemistry, demography, economics, geoscience, mathematical psychology, probability, sociology, political science. A generalization of the logistic function is the hyperbolastic function of type I. The logistic differential equation is often represented in the following manner:

$$\int \frac{dP}{P(1-\frac{P}{K})} = \int k.\,dt$$

1.2. Review of literature: A research paper from Live Lab proposed a methodology to measure real-world smartphone usage with a reprogrammable in device logger where an iPhone 3Gs was deployed to 25 users for 1 year to collect data and test the potential of the methodology used to gather the data. The preliminary data suggests the potential it needed obtain the data but it lacked in large scale data gathering, which in today's world becomes a privacy issue due to the various laws and regulations put forth by different nations around the world. Which becomes far more complicated than a simple mathematical equation to estimate the number of active smartphone users [1].

2.1. Problem Statement: to find the estimated number of active smartphone users, the prediction which can explain to us how the pandemic has begun to replace the outdoor activity to a digital platform. The reason for this study is to urge the many business and other educational platform to adopt the digital platform as the rise in numbers of the population on their digital presence.

2.2. Theory: The General logistic function is the solution of the simple first order non-linear ordinary differential equation: (refer Logistic curve 1 for example in 2.6)

$\frac{d}{dx}f(x) = f(x)\big(1 - f(x)\big)$ With boundary condition: $f(0) = \frac{1}{2}$. It can be understood as: the derivative is 0 when the function is 1.The derivative is positive for f between 0 and 1, and negative for f above 1 or less than 0. This results in an unstable equilibrium at 0 and a stable equilibrium at 1, and thus for any function value greater than 0 and less than 1, it increases to 1.

The logistic differential equation is the special case of the Bernoulli differential equation and has the following solution: $f(x) = \frac{e^x}{(e^x - C)}$To estimate the number of smartphone users in India with a population of 1.381 Billion we acquire the data of the smart phone users of the years 2015-2017 with the data of just two consecutive years and find the estimated smartphone users in 2020, and further. Since the population growth in India is exponential, we need to convert the Differential Equation to an Analytical solution in an exponential form, in this case the logistic differential equation can only be changed using the variable Separation method to obtain the required Exponential Equation:

$$\frac{dP}{dt} = k.P\left(1 - \frac{P}{K}\right) => \int \frac{dP}{P(1-\frac{P}{K})} = \int k.dt$$

Where P= Population of Smartphone users; K=Total population capacity; t=time (in years); k=Constant. In order to evaluate the LHS we write:

$$\frac{1}{P\left(1-\frac{P}{K}\right)} = \frac{K}{P(K-P)} = \frac{1}{P} + \frac{1}{K-P}$$

$$\int \frac{dP}{dP} + \int \frac{dP}{(K-P)} = \int k.dt$$

$$\ln|P| - \ln|K-P| = kt + C$$

$$\ln\left|\frac{K-P}{P}\right| = -kt - C$$

$$\left|\frac{K-P}{P}\right| = e^{-kt-C}$$

$$\frac{K-P}{P} = Ae^{-kt}\ ;\ (A = \pm e^{-C})$$

$$P = \frac{K}{1+Ae^{-kt}} \quad Where\ A = \frac{K-P_0}{P_0}$$

2.3. Data: (in terms of millions) Indian Population (2020) K=1381.9M; Initial smartphone users (2015) P_0 =250.66M; Smartphone users (2016) P_1=304.51M; Smartphone users in (2020) P_5= ? [2]

2.4. Analysis-I: we input the data in the above equation 1 and proceed further. We get:

$$\boxed{P_t = \frac{1381.9}{1+(4.513046)e^{-kt}} \text{ ---- (2)}};\ where\ A = \frac{1381.9-250.66}{250.66} = \frac{1131.24}{250.66} = 4.513046$$

To find k we make use of the data from P_1 =304.51 in equation –(1);we get-

$$P_1 = \frac{1381.9}{1+(4.513046)e^{-k}} = 304.51$$

upon further Simplification- $\frac{1}{1+(4.513046)e^{-k}} = 0.22035\epsilon$

$$(4.513046)e^{-k} = \frac{1}{0.220356} - 1$$

$$e^{-k} = \frac{3.538111}{4.51304\epsilon}$$

$$\ln(e^{-k}) = \ln\left(\frac{3.538111}{4.513046}\right)$$

$$-k = \ln\left(\frac{3.538111}{4.513046}\right) \text{----(3)}$$

Now substituting the value of -k in equation 2 we obtain the following:

$$P_t = \frac{1381.9}{1 + (4.513046)e^{\ln\left(\frac{3.538111}{4.513046}\right)*t}}$$

$$\boxed{P_t = \frac{1381.9}{1+(4.513046)\left(\frac{3.538111}{4.513046}\right)^t} \text{-----(4)}}$$

We have now arrived at Standard solution, to calculate the exponential growth in 5 years i.e. (2020) we input t=5.

$$P_5 = \frac{1381.9}{1 + (4.513046)\left(\frac{3.538111}{4.513046}\right)^5}$$

$$P_5 = \frac{1381.9}{1 + \frac{(3.538111)^5}{(4.513046)^4}}$$

$$P_5 = \frac{1381.9}{1 + 1.33652\text{9}}$$

$$P_5 = \frac{1381.9}{2.336529}$$

$P_5 = 591.432848M$ ---(5)

Similarly, we can calculate from P_0 *to* P_{10}. We get the following: (in millions)

$P_1 = 304.509960; P_2 = 366.183865;\ P_3 = 435.302054;\ P_4 = 512.904240$

$P_5 = 591.432848;\ P_6 = 674.820545;\ P_7 = 758.680891;\ P_8 = 840.573716;$

$P_9 = 918.28144;\ P_{10} = 990.034503;\ P_{96} = 1381.900000$(Refer logistic curve-2 in 2.6)

2.5.Analysis-II: the data obtained in P_5 is accurate according to the exponential growth using the logistic differential equation but a major variable has been added in the year 2020 which makes the difference as to why the there is a significant rise in the smartphones. The Variable being the Pandemic (COVID-19) which forced the nation to stay indoors and self-quarantine which resulted in a large-scale digitalization in the nation. Most of the nation adapted quickly to the new situation and shifted their work remotely and on a digital platform. As digitalization became the new trend and most of the nation using a digital platform for their work, the nation saw a huge rise in sales of smartphones in its 2nd Quarter and the sales recorded where found to be at 43% more than the active smartphone users in 2015 according to a research conducted by Counterpoint [3]. This variable brings a slight change in the overall estimate of 2020 which becomes: (refer logistic curve-3 in 2.6)

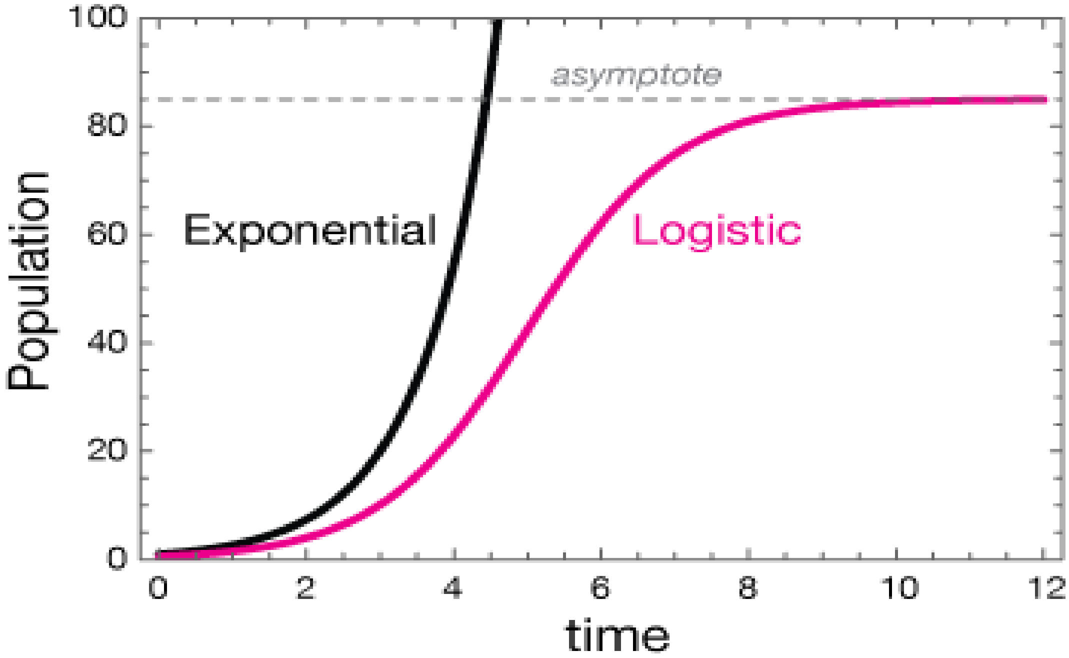
43% of $P_0 = \frac{(43*250.66)}{100} => P_0' = 107.783791\ M$

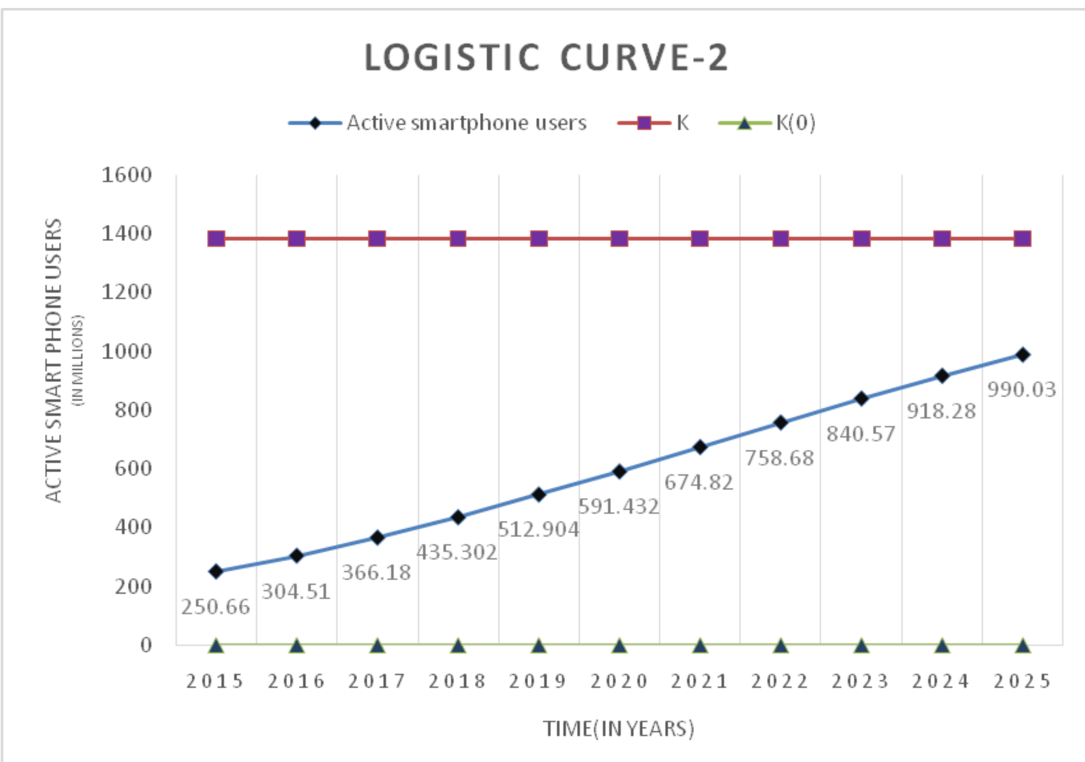
Now, the accurate Estimate of 2020 (P_5') = $P_0' + P_5$

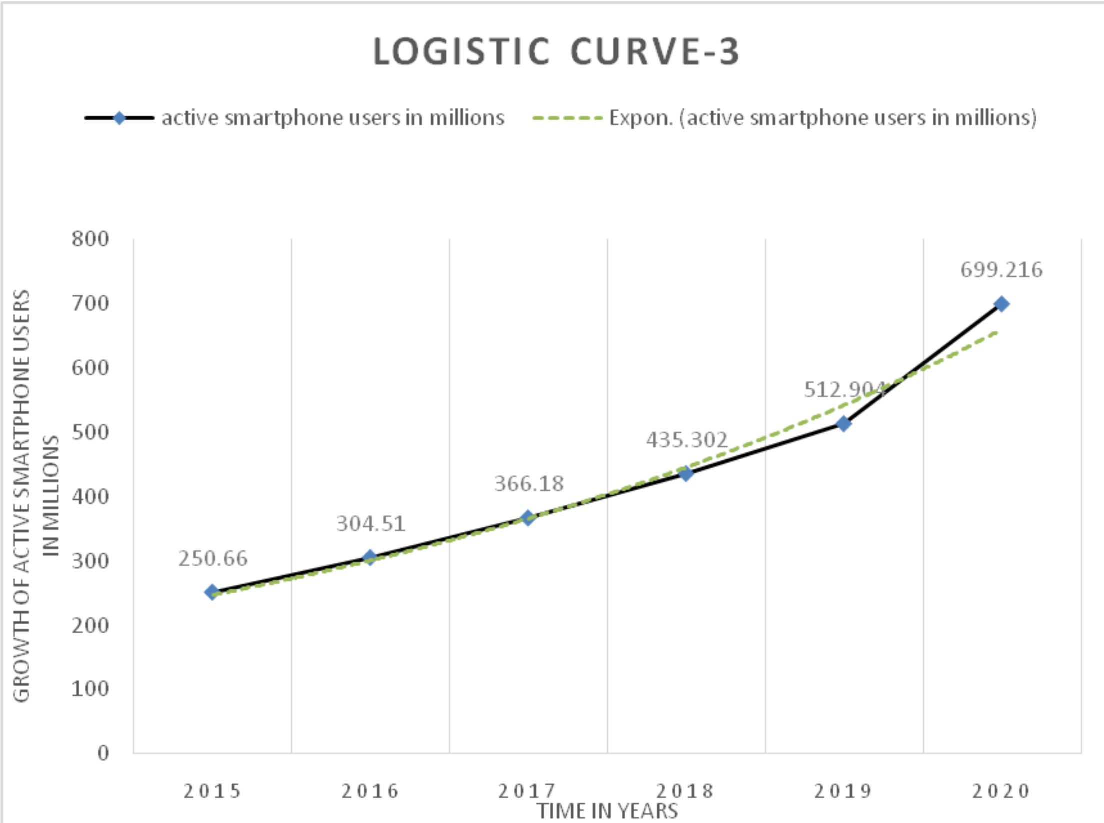
$P_5' = 591.432848 + 107.783791$

$$\boxed{P_5' = 699.216639\ M}$$

2.6.Data Interpretation: (charts/graphs) Logistic Curve -1







3. Findings: The Growth in active smartphone users was found using the logistic Differential equation. The data as to why it's important now is that during a pandemic the nation tends to meet the digital frontier quickly than ever before, with the data from above we as a nation must encourage others to also shift to a digital platform as the digital web presence of a person now grows drastically. With the dream of our PM of digitalizing India called digital India is now meeting its goals at a much higher rate, we can accomplish may things like, High speed internet to rural areas at a low cost, increase in online transactions and much more.

4. Scope of the study: The Scope of the study was to find the growth of smartphones and an increase in the number of active users, with this data we can seize the opportunity to:

- Optimize the digital platform on smartphones.
- Promote more online transactions.
- Increase Mobility of Job.
- Shift most of the physical work into digital platforms.
- Increase the number of cloud computing software and optimize them.

4.1. Conclusions: The logistic curve was found and estimated the active smartphone users in India. The 699Million Active Smartphone users were found due to the pandemic as most shifted their business to a digital platform.

4.2. Strengths: As the web-presence of the citizens grows, this is the greatest boon to all marketers as they can place ads on many digital platforms at a very low cost, it also helps a lot of small business to build their web presence. The main key feature of going digital is that all schools, colleges, business can stay safe at home during the pandemic and also get the job done. The social network of the nation also gets closer as the citizens of the nation [4].

4.3. Weakness: Unsecure Web platforms, lots of data consumption, increase in hacking, mental health degradation, loss of human contact, uneducated smartphone users susceptible to fraud, Rise in misinformation, damage to eyes .[5]

4.4. Opportunities:

- Online Schools and classes.
- Online Business.
- Online Marketing.
- Social networks.

4.5. Threats:

- Lack of trust on Digital platform
- Mental health degradation.
- Loss of human contact and social activities.
- Phishing.

5. Images:

Figure 1: phishing

Figure 2: UPI Payments

Figure 3: Online classes via Microsoft teams

Figure 4: Digital India

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14. APPLICATION OF FIBONACCI SEQUENCE IN DAY TODAY LIFE

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Abstract: Fibonacci sequence of numbers and th**e associated "Golden Ratio" are** manifestedinnatureandincertainworksofart.Weobservethatmanyofthenaturalthingsfollow the Fibonacci sequence. It appears in biological settings such as branching in trees, phyllotaxis (the arrangement of leaves on a stem), the fruit sprouts of a pineapple, the flowering of an artichoke, an uncurling fern and the arrangement of a pine cones bracts etc. At present Fibonacci numbers plays very important role in coding theory. Fibonacci numbers in different forms are widely applied in constructing security coding.

Keywords: Fibonacci Numbers, Golden ratio, Coding, Encryption, Decryption etc.

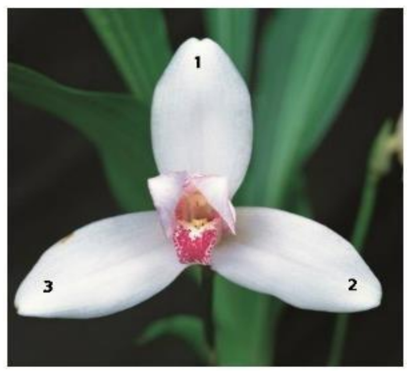
Introduction: TheFibonaccinumberswerefirstdiscoveredbyamannamedLeonardoPisano. He was known by his nickname, Fibonacci. The Fibonacci sequence is a sequence in which each term is the sum of the 2 numbers preceding it. The Fibonacci Numbers are defined by the recursive relation defined by the equations Fn = Fn-1 + Fn-2 for all n ≥ 3 where F1 = 1; F2 = 1 where Fn represents the nth Fibonacci number (n is called an index).The Fibonacci sequence can elaborately written as,**{**1,1,2,3,5,8,13,21,34,55…….}.One of the most common experiments dealing with the Fibonacci sequence is his experiment with rabbits. Fibonacci put one male and one female rabbit in a field. Fibonacci supposed that the rabbits lived infinitely and every month a new pair of one male and one female was produced. Fibonacci asked how many would be formed in a year. Following the Fibonacci sequence perfectly the rabbits reproduction was determined...144 rabbits. Though unrealistic, the rabbit sequence allows people to attach a highly evolved series of complex numbers to an everyday, logical, comprehendible thought. Bortner and Peterson elaborately described the history and application of Fibonacci numbers.

Now let' us see the application of Fibonacci sequence: Tree—we see them everywhere, but do you look and analyse the structure of how the branches grow out of the tree and each other? No, because you're normal and have better things to do. But if you did, you would see the Fibonacci sequence evolve out of the trunk and spiral and growth. Fibonacci can be found in nature not only in the famous rabbit experiment, but also in beautiful flowers. On the head of a sunflower and these eds are packed in a certain way so that they follow the pattern of the Fibonacci sequence. This spiral prevents the seed of the sunflower from crowding themselves out, thus helping them with survival. The petals of flowers and other plants may also be related to the Fibonacci sequence in the way that they create new petals.

Petals on flowers Probably most of us have never taken the time to examine very carefully the number or arrangement of petals on a flower. If we were to do so, we would find that the number of petals on a flower that still has all of its petals intact and has not lost any, for many flowers is a Fibonacci number.

The number of petals on a flower is often one of the Fibonacci numbers. For example,

- Two-petalled flowers are those of Crown oft horns.
- Three-petalled flowers are quite common, viz., trillium and iris. There are hundreds of species, both wild and cultivated, with five petals, viz ..We can observe inflower’s:



Even we can observe the application of Fibonacci sequences in leaves arrangements of plants

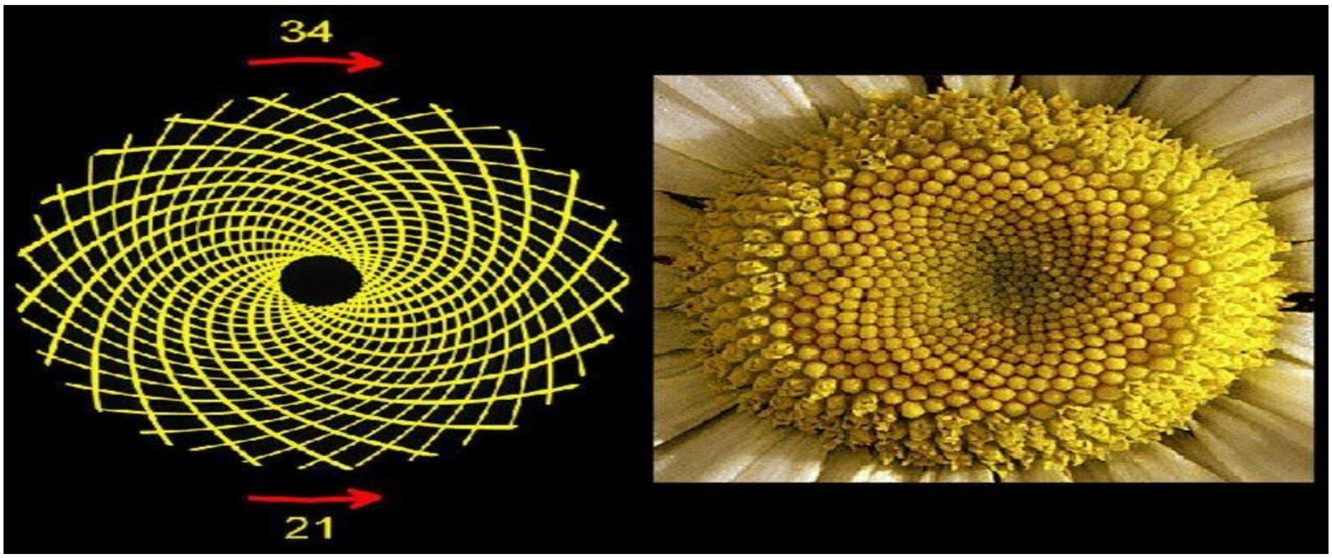
Many plants show the Fibonacci numbers in the arrangements of the leaves around their stems. If we look down on a plant, the leaves are often arranged so that leaves above do not hide leaves below. This means that each gets a good share of the sunlight and catches the most rain to channel down to the roots as it runs down the leaf turn.

Leaves per turn. The Fibonacci numbers occur when counting both the number of times we go around the stem, going from leaf to leaf, as well as counting the leaves we meet until we encounter a leaf directly above the starting one. If we count in the other direction, we get a different number of turns for the same number of leaves. The below are computer-generated "plants", but you can see the same thing on real plants. One estimate is that 90 percent of all plants exhibit this pattern of leaves involving the Fibonacci numbers. Some common trees with their Fibonacci leaf arrangement numbers visible are, elm, linden, lime, grasses, beech, hazel, grasses, blackberry, oak, cherry, apple, holly, plum, common groundsel, poplar, rose, pear, willow, pussy willow, almond where n/t means there are n leaves in t turns or n/t leaves per turn. Cactus's spines often show the same spirals as we have already seen on pine cones, petals and leaf arrangements, but they are much more clearly visible. Let us have a look over arrangements of leaves:



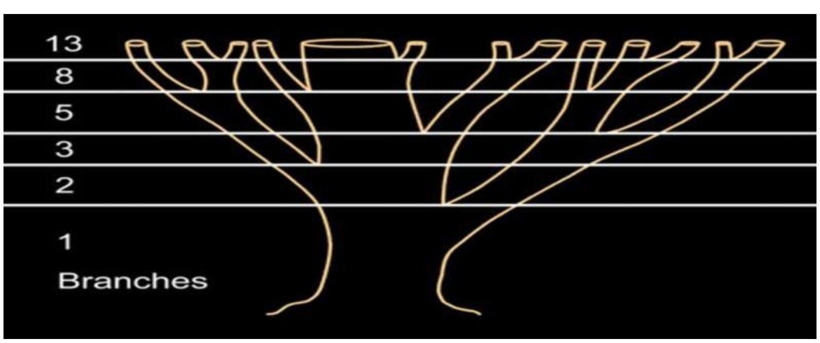


Seed heads: The head of a flower is also subject to Fibonaccian processes.Typically, seeds are produced at the center, and then migrate towards the outside to fill all the space. Sunflowers provide a great example of these spiraling patterns.



In some cases, the seed heads are so tightly packed that total number can get quite high as many as 144 or more. And when counting these spirals, the total tends to match a Fibonacci number. Interestingly, a highly irrational number is required to optimize filling (namely one that will not be well represented by a fraction). Phi fits the bill rather nicely.

Fruits and Vegetables: Likewise, similar spiraling patterns can be found on pineapples and cauliflower.

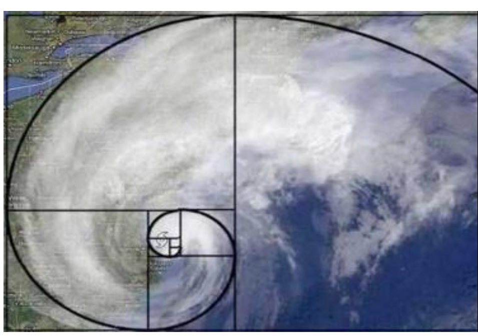
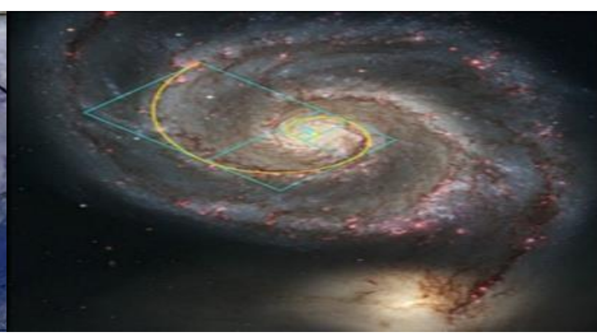


Tree branches: The Fibonacci sequence can also be seen in the way tree branches form or split. A main trunk will grow until it produces a branch, which creates two growth points. Then, one of the new stems branches into two, while the other one lies dormant. This pattern of branching is repeated for each of the new stems. A good example is the sneezewort. Root systems and even algae exhibit this pattern.

In shells: The unique properties of the Golden Rectangle provides another example. This shape, a rectangle in which the ratio of the sides a/b is equal to the golden mean (phi), can result in a nesting process that can be repeated into infinity—and which takes on the form of a spiral. Its call the logarithmic spiral, and it a bounds in nature. Snail shells and nautilus shells follow the logarithmic spiral, as does the cochlea of the inner ear. It can also be seen in the horns of certain oats, and the shape of certain spider's webs.

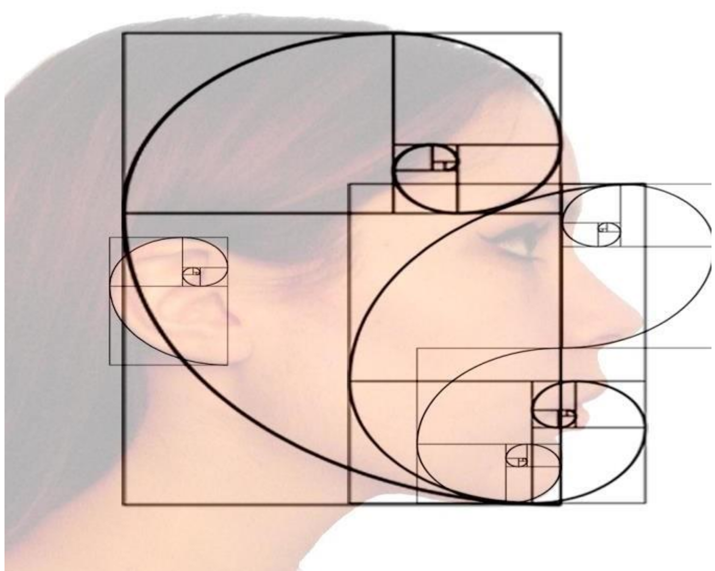
Spiral galaxy: The Milky Way has several spiral arms, each of them a logarithmic spiral of about12degrees. As an interesting aside, spiral galaxies appear to defy Newtonian physics. As early as 1925, astronomers realized that, since the angular speed of rotation of the galactic disk varies with distance from the center, the radial arms should become curved as galaxies rotate. Subsequently, after a few rotations, spiral arms should start to wind around a galaxy. But they don't—hence the so-called winding problem. The stars on the outside, it would seem, move at a velocity higher than expected — a unique trait of the cosmos that helps preserve its shape.

Storms: Specifically hurricanes and tornadoes, many storm systems follow the Fibonacci Sequence. I suppose this is not beautiful, but more interesting. On a map, at least, hurricanes look cool. I guess we could say this example proves math can be beautiful and destructive.

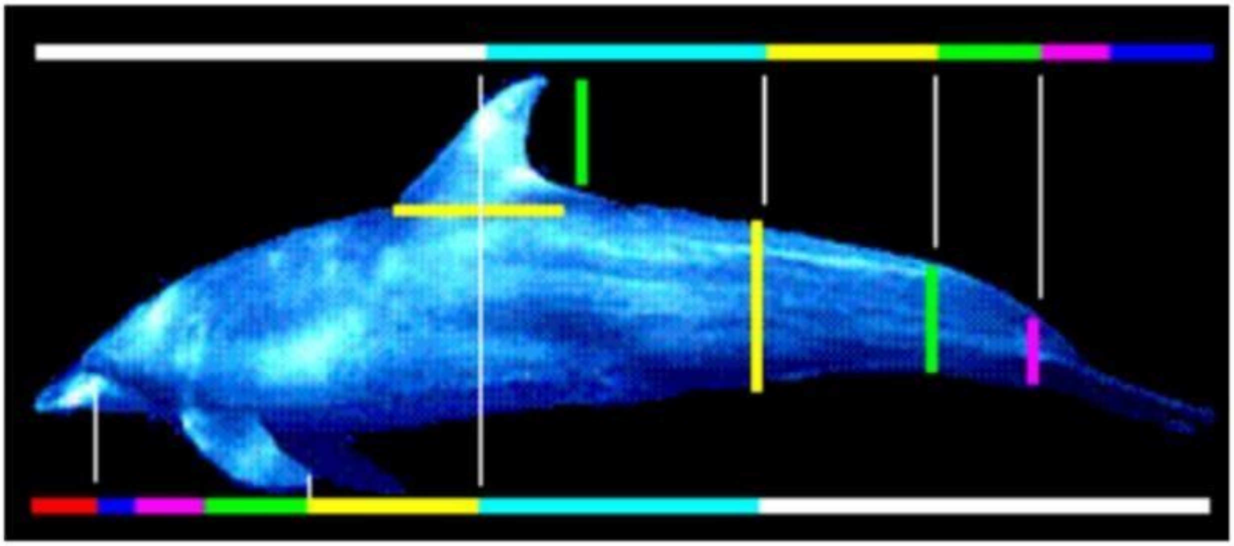




Fibonacci sequences in human body: The same phenomena of Phi that is found in nature's objects from snail shells to the spirals of galaxies is found also in the design and structure of the human body. For example, the cochlea of the ear is a Fibonacci spiral as is the spiral of the umbilical cord. The progression of the Fibonacci numbers and ratio are well suited to describing organic growth in the human body because they have the properties of self-similarity and of "gnomonic growth;" that is, only the size changes while the shape remains constant. The majority of organs in the human body maintain their overall shape and proportions as they grow.

The Human Face: Faces, both human and non-human, a bound with examples of the Golden Ratio. The mouth and nose are each positioned at golden sections of the distance between the eyes and the bottom of the echin. Similar proportions can been seen from the side, and even the eye and ear itself (which follows along a spiral). It's worth noting that every person's body is different, but that averages across populations tend towards phi. It has also been said that the more closely our proportions adhere to phi, the more "attractive" those traits are perceived. As an example, the most "beautiful" smiles are those in which central incisors are 1.618 wider than the lateral incisors, which are 1.618 wider than canines, and soon. It's quite possible that, from anevo-psychperspective, that we are primed to like physical forms that adhere to the golden ratio—a potential indicator of reproductive fitness and health. Many features of the “ideal” human face are said to have ratios equal to φ; the dimension relationships between the eyes, ears, mouth and nose, for instance. The ratio of the height of the whole head to that of the head above the nose is also said to be Phi. Other examples supposedly include the ratio between the total height of the body and the distance from head to the fingertips, and “the distances from head to naval and naval to hill.” Then there is the proportion between the forearm and upper arm and the one between the hand and forearm; all of these are said to follow the rule of Golden Ratio.



Animal bodies:



Even our body's exhibit proportions that are consistent with Fibonacci numbers. For example, the measurement from the navel to the floor and the top of the head to the navel is the golden ratio. Animal bodies exhibit similar tendencies, including dolphins (the eye, fins and tail all fall at Golden Sections), starfish, sand dollars, sea urchins, ants, and honeybees.

Reproductive dynamics: Speaking of honey bees, they follow Fibonacci in other interesting ways. The most profound example is by dividing the number of females in a colony by the number of males (females always out number males). The answer is typically something very close to 1.618. In addition, the family tree of honey bees also follows the familiar pattern. Males have one parent (a female), whereas female shave two (a female and male). Thus, when it comes to the family tree, males have 2, 3, 5, and 8 grandparents, great-grandparents, gr-gr-grandparents, and gr-gr-gr-grandparents respectively. Following the same pattern, females have 2, 3, 5,8,13, and so on. And as noted, bee physiology also follows along the Golden Curve rather nicely.

DNA molecule: Even the microscopic realm is not immune to Fibonacci. The DNA molecule measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix spiral. These numbers, 34 and 21, are numbers in the Fibonacci series, approximately Phi, 1.6180339.

Other applications of Fibonacci sequence:

- The Fibonacci numbers are also an example of a complete sequence. This means that every positive integer can be written as a sum of Fibonacci numbers, where any one number issued once at most. Moreover, every positive integer can be written in a unique way as the sum of *one or more* distinct Fibonacci numbers in such a way that the sum does not include any two consecutive Fibonacci numbers. This is known as Zeckendorf's theorem, and a sum of Fibonacci numbers that satisfies these conditions is called a Zeckendorf's representation. The Zeckendorf's representation of a number can be used to derive its Fibonacci coding.
- Fibonacci numbers are used by some pseudo random number generators.
- They are also used in planning poker, which is a step in estimating in software development projects that use the Scrum methodology.
- Fibonacci numbers are used in a polyphone version of the merge sort algorithm in which an unsorted list is divided into two lists whose lengths correspond to sequential Fibonacci numbers – by dividing the lists that the two parts have lengths in the approximate proportion φ. A tape-drive implementation of the polyphas emerge sort was described in *The Art of Computer Programming*.
- Fibonacci numbers arise in the analysis of the Fibonacci heap data structure.
- The Fibonacci cube is an undirected graph with a Fibonacci number of nodes that has been proposed as a network topology for parallel computing.
- A one-dimensional optimization method, called the Fibonacci search technique, uses Fibonacci numbers.
- The Fibonacci number series is used for optional lossy compression in the IFF8SVX audio file format used on Amiga computers. The number series compands the original audio wave similar to logarithmic methods such asμ-law.[25][26]\
- Since the conversion factor 1.609344 for miles to kilometres is close to the golden ratio, the decomposition of distance in miles into a sum of Fibonacci numbers becomes nearly the kilometre sum when the Fibonacci numbers are replaced by their successors. This method

amounts to a radix 2 number register in golden ratio base φ being shifted. To convert from kilometres to miles, shift the register down the Fibonacci sequence instead.

- In optics, when a beam of light shines at an angle through two stacked transparent plates of different materials of different refractive indexes, it may reflect off three surfaces: the top, middle, and bottom surfaces of the two plates. The number of different beam paths. That have k reflections, for $k > 1$, is the kth Fibonacci number. (However, when $k = 1$, there are three reflection paths, not two, one for each of the three surfaces.)

Conclusion: The Fibonacci numbers are Nature's numbering system. They appear everywhere in Nature, from the leaf arrangement in plants, to the pattern of the florets of a flower, the bracts of a pinecone, or the scales of a pineapple. The Fibonacci numbers are therefore applicable to the growth of every living thing, including a single cell, a grain of wheat, a hive of bees, and even all of mankind. Nature follows the Fibonacci number astonishingly. But very little we observe the beauty of nature. The Great poet Rabindranath Tagore also noted this. If we study the pattern of various natural things minutely we observe that many of the natural things around us follow the Fibonacci numbers in real life which creates strange among us. The study of nature is very important for the learners. It increases the inquisitiveness among the learners. The topic is chosen so that learners could be interested towards the study of nature around them. .Security in communication system is an interesting topic at present as India is going towards digitalizati